Parameter estimation of physical models for application in building control

K.U.Leuven, Belgium

Department of Mechanical Engineering

Lieve Helsen, Clara Verhelst

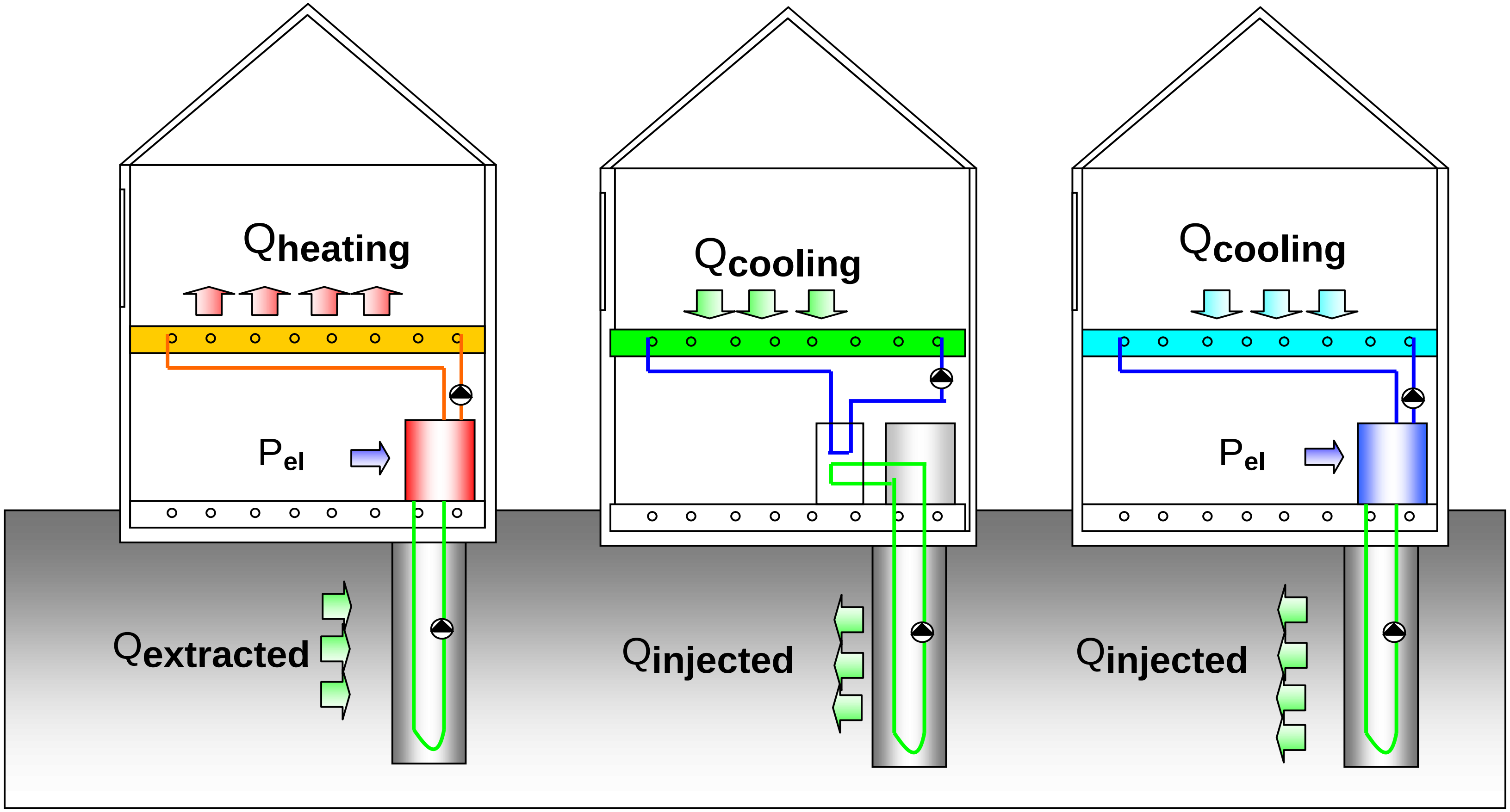
Outline

- Introduction
- Framework of Model Predictive Control
- Development of control relevant model
- Applications in building control

INTRODUCTION

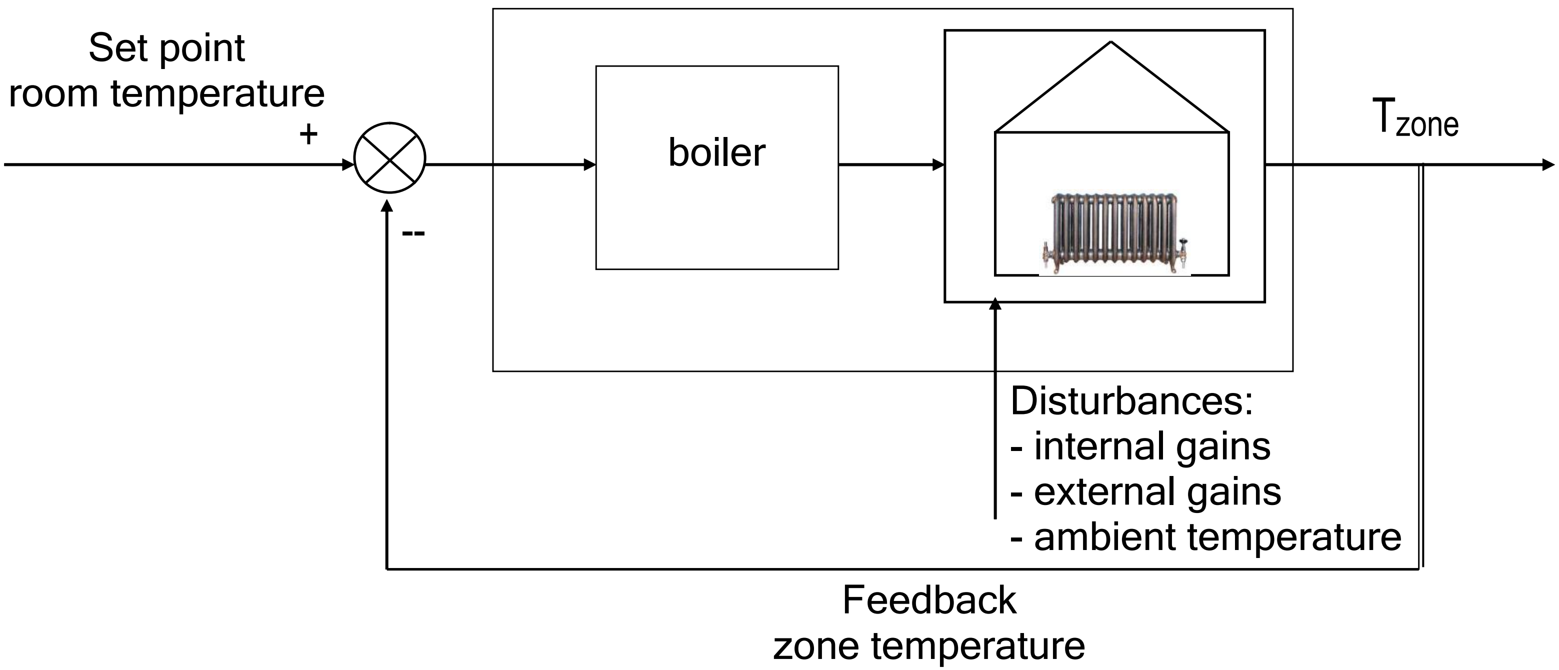
Introduction

- Model predictive control (MPC) of ground coupled heat pump systems



Introduction

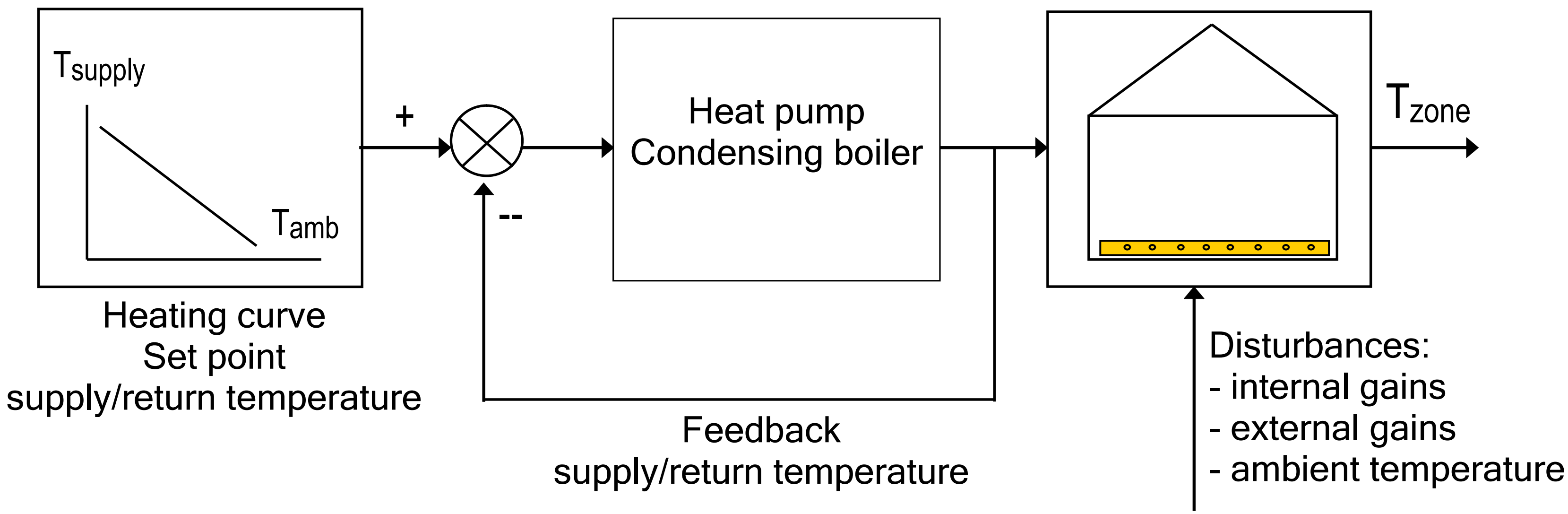
- Control of traditional heating systems



Room thermostat

Introduction

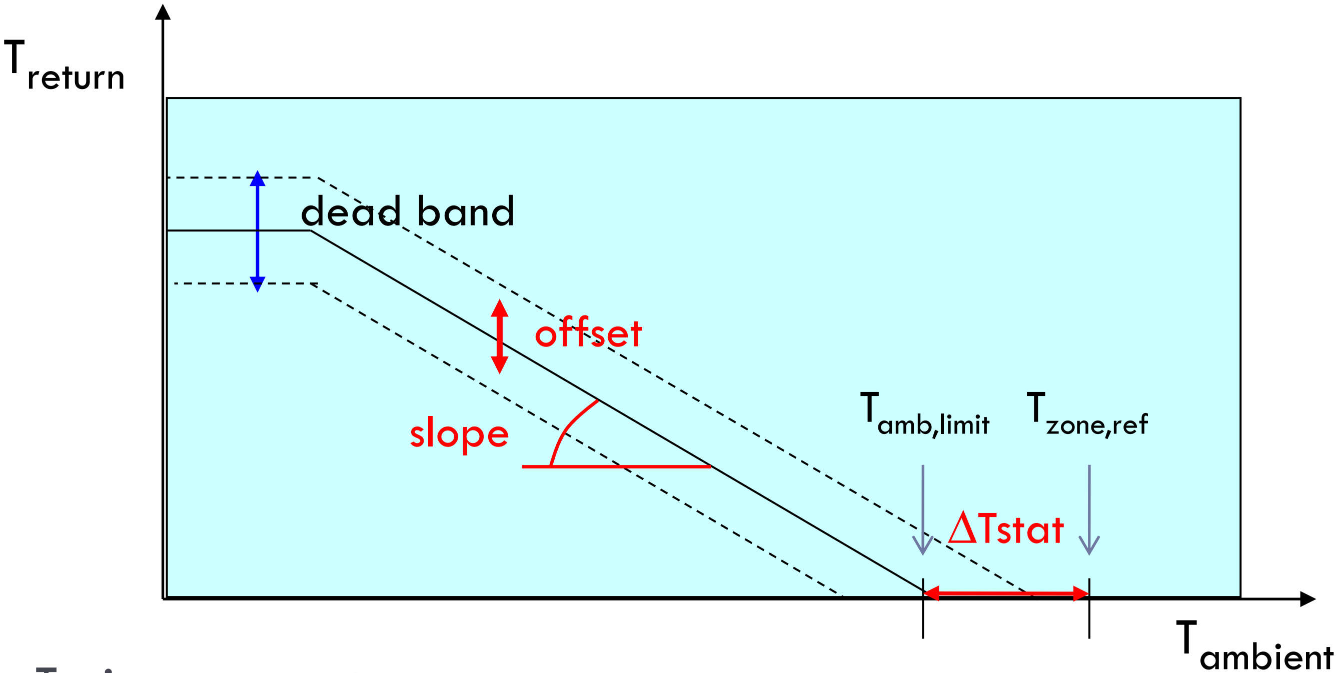
- Control of low temperature heating systems



Heating curve

Introduction

- Heating curve ~ static building model



Tuning parameters:

- Slope proportional to heat losses = UA (T_{room} – $T_{ambient}$)
- Offset depends on set point T_{room}
- ΔTstat depends on heat gains (internal and solar)

- Control of heating & cooling systems

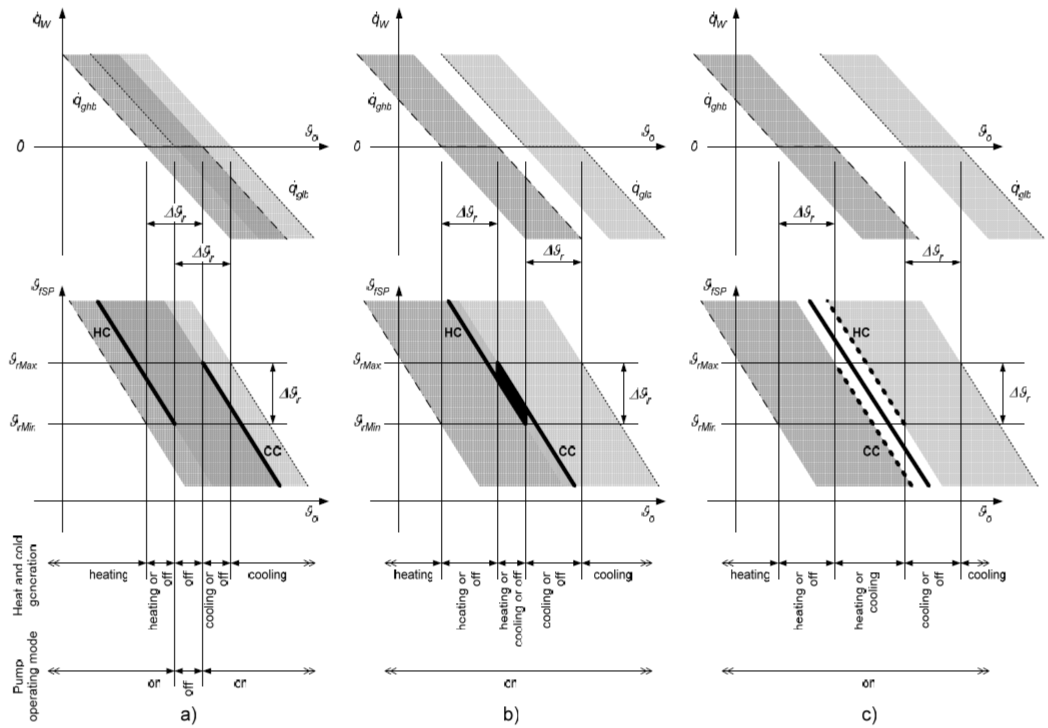


Figure 6. Different uncertainties in heat gains: a) Low, b) Medium, c) High.

Rule based control

Source: Werner et al., 2005

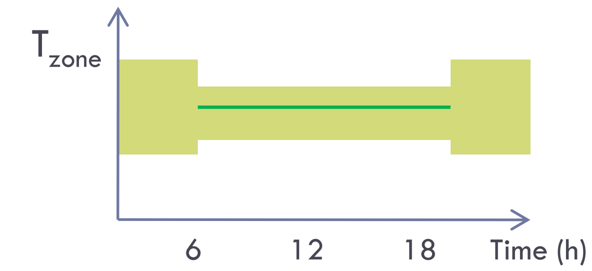
Introduction

- Traditional control hierarchy

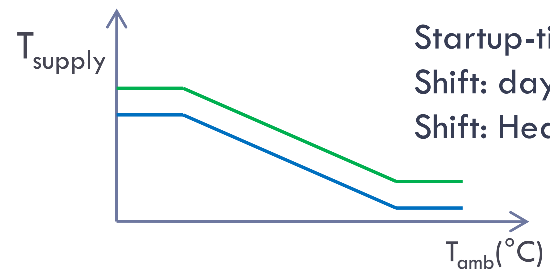
Setpoint optimization at plant level

Static optimization

Setpoint optimization at unit level



Logics (if-then-else)
Exception handling
Ad hoc solutions



Startup-time
Shift: day-night
Shift: Heating-cooling

Local Loop Controllers

PID –control T_{supply}

Actuators

Closed-loop stable

Compressor power
Circulation pumps
Switching valves

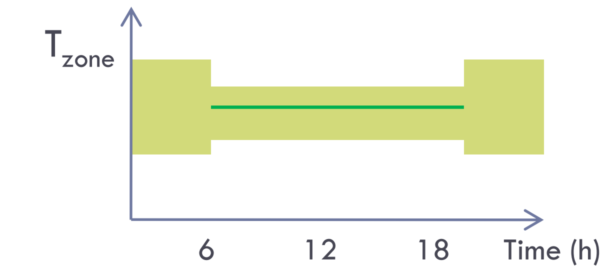
Source: Maciejowski 2002

- MPC in the control hierarchy

Setpoint optimization at plant level

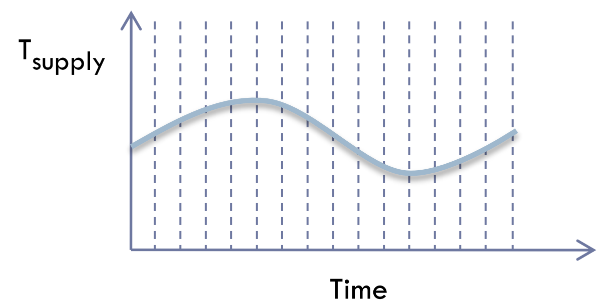
Static optimization

Setpoint optimization at unit level



Model Predictive Control

Dynamic optimization



Local Loop Controllers

PID –control T_{supply}

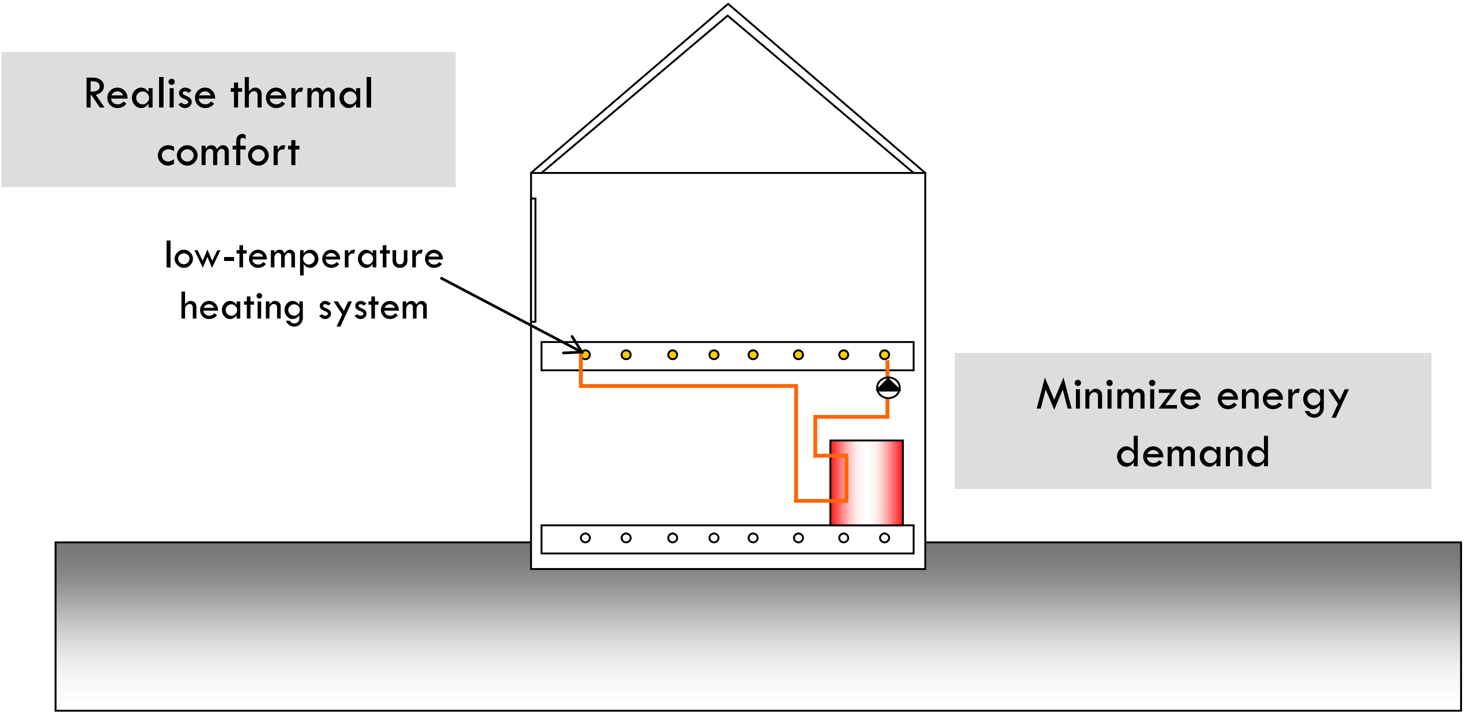
Actuators

Closed-loop stable

Compressor power
Circulation pumps
Switching valves

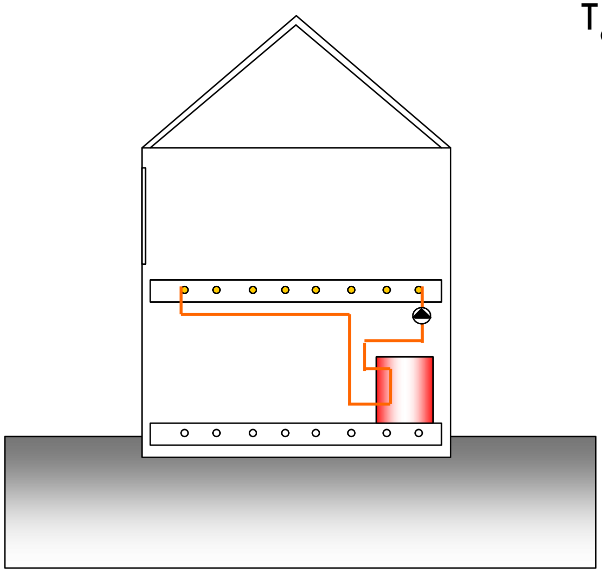
Source: Maciejowski 2002

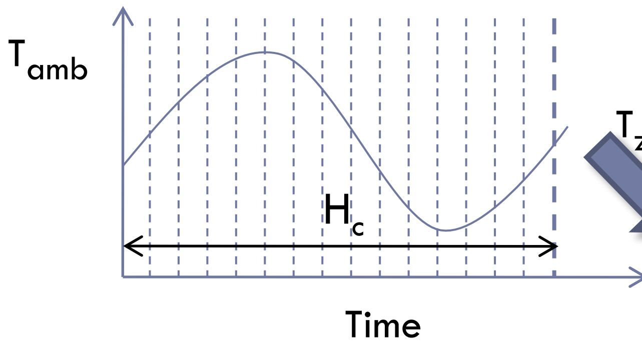
- Example 1: Condensing boiler

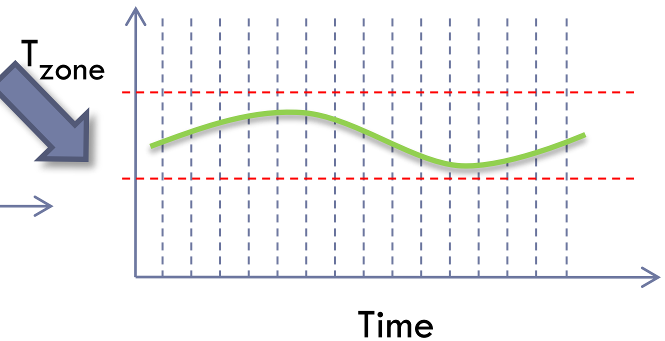


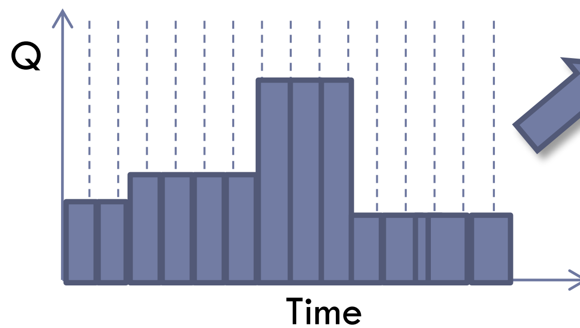
Optimal control objectives

- Example 1: Condensing boiler









Building model

$T_{zone} = f(T_{amb}, Q)$

Optimal control formulation

$$\dot{Q}^* = \underset{u}{\arg\min} \int_0^{Hc} \dot{Q}(t)\,dt$$

subject to:

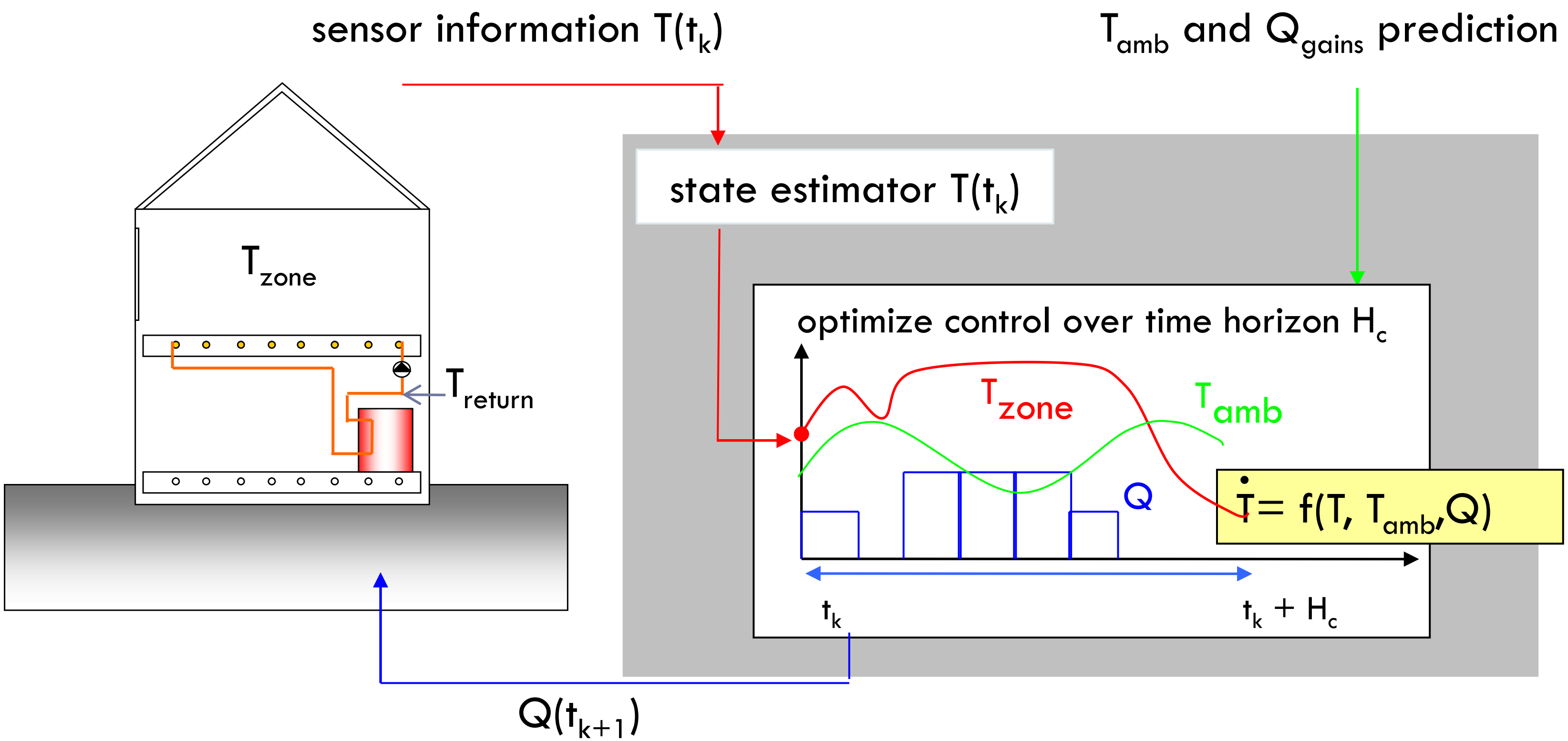
$$\dot{T}_{zone} = f(T_{zone}, T_{amb}, \dot{Q})$$

$$T_{\min}(t) \le T_{zone}(t) \le T_{\max}(t)$$

$$0 \le \dot{Q}(t) < \dot{Q}_{\max}$$

Introduction

- Example 1: Condensing boiler

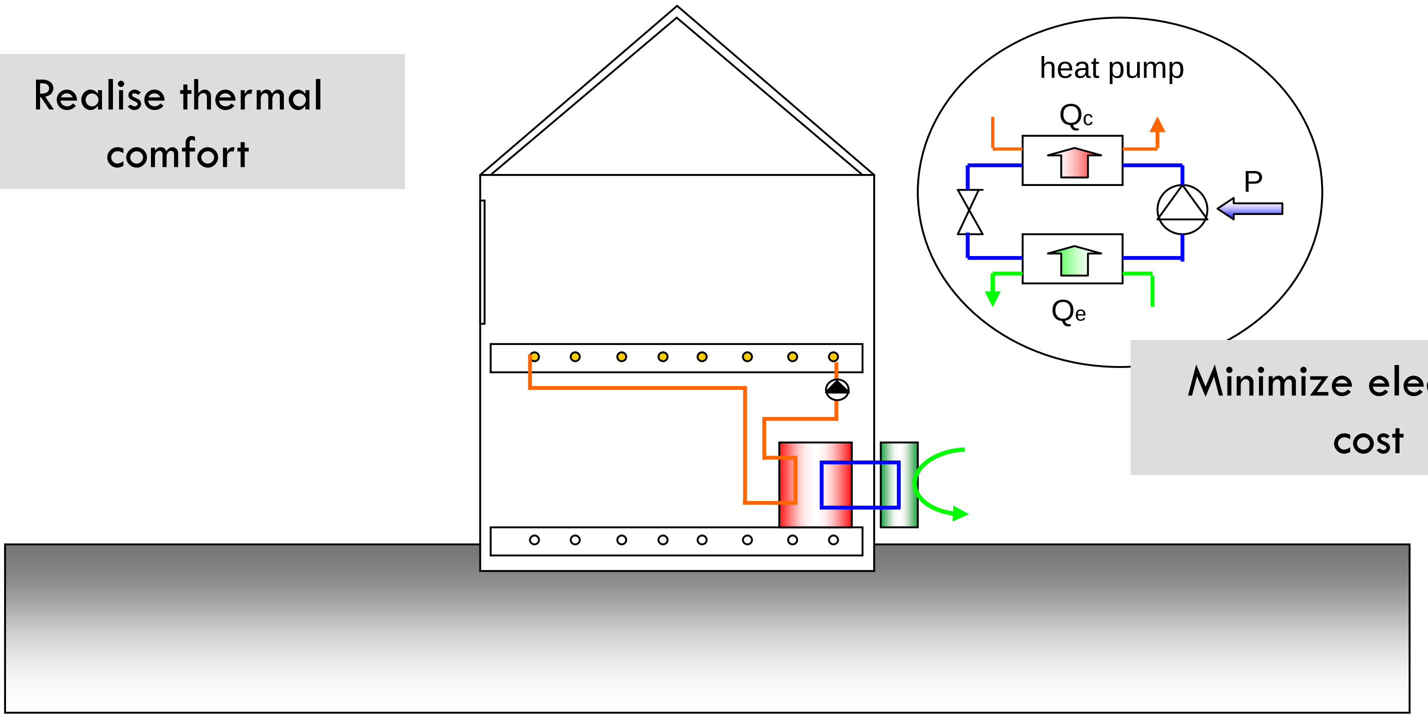


Model Predictive Control scheme

Introduction

- Example 2: Air-to-water heat pump system

Realise thermal comfort



Minimize electricity cost

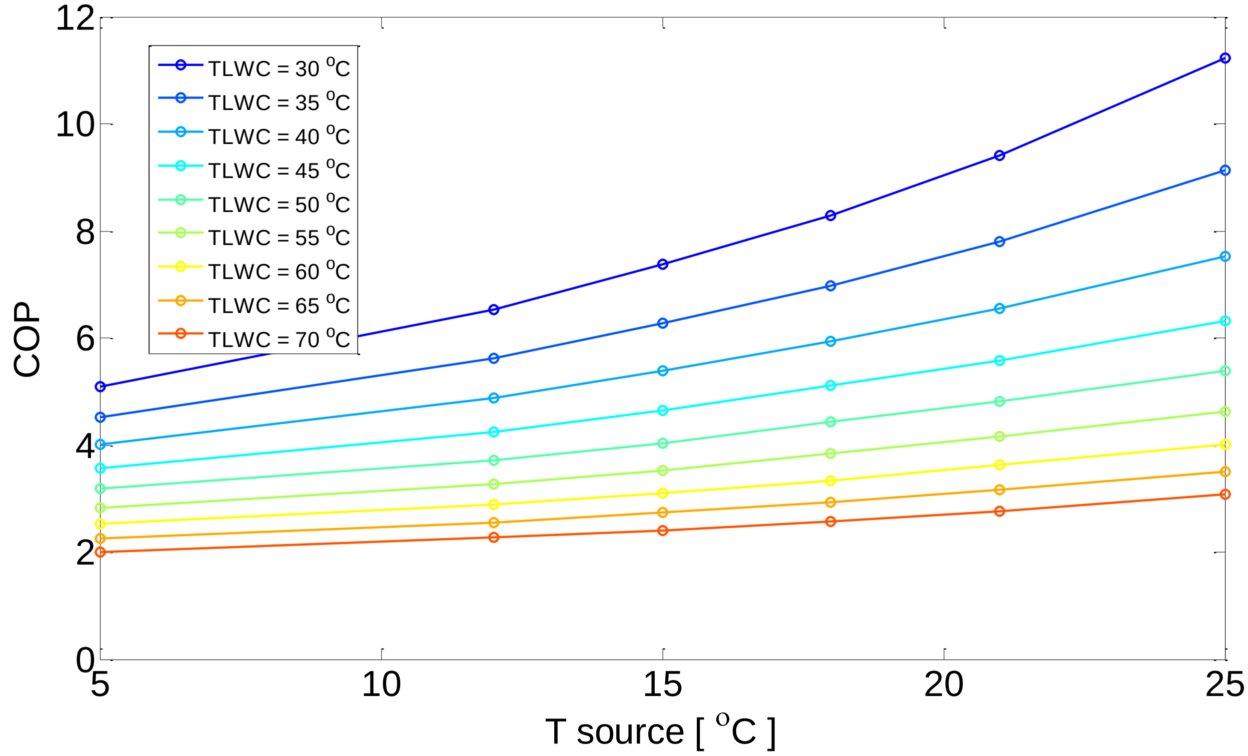
Optimal control objectives

$$COP = \frac{\dot{Q}_c}{P} = f\left(T_{amb}, T_{supply}\right)$$

Coefficient of performance (COP)

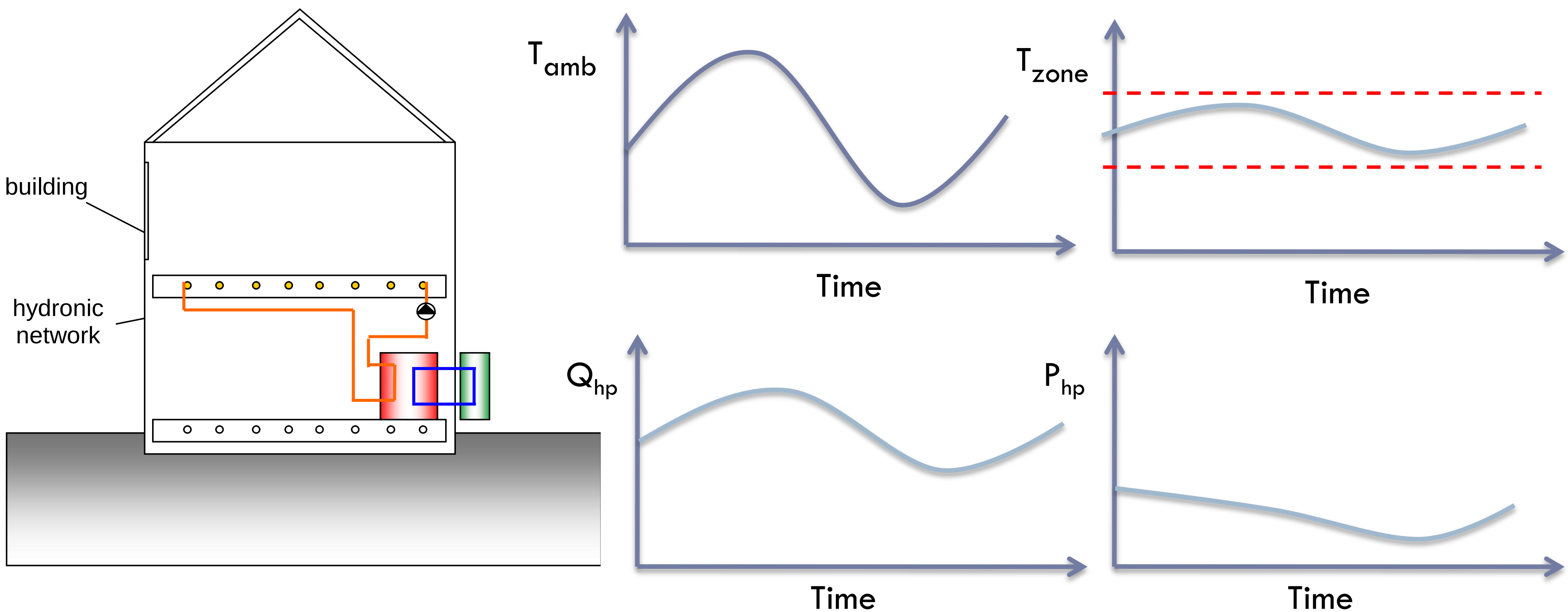
$$COP = \frac{\dot{Q}_{supply}}{P_{compressor}}$$

$$COP_{\max} = \frac{T_{supply}}{T_{supply} - T_{source}}$$

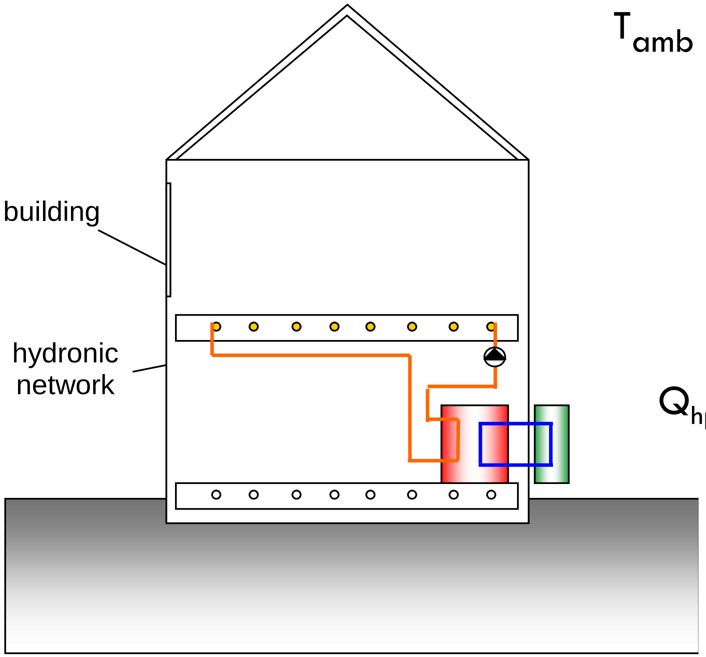


$$COP \approx (c_0 + c_1 T_{source} + c_2 T_{supply})$$

- Example 2: Air-to-water heat pump system
  - Guaranteed thermal comfort
  - Minimal electricity demand



Introduction

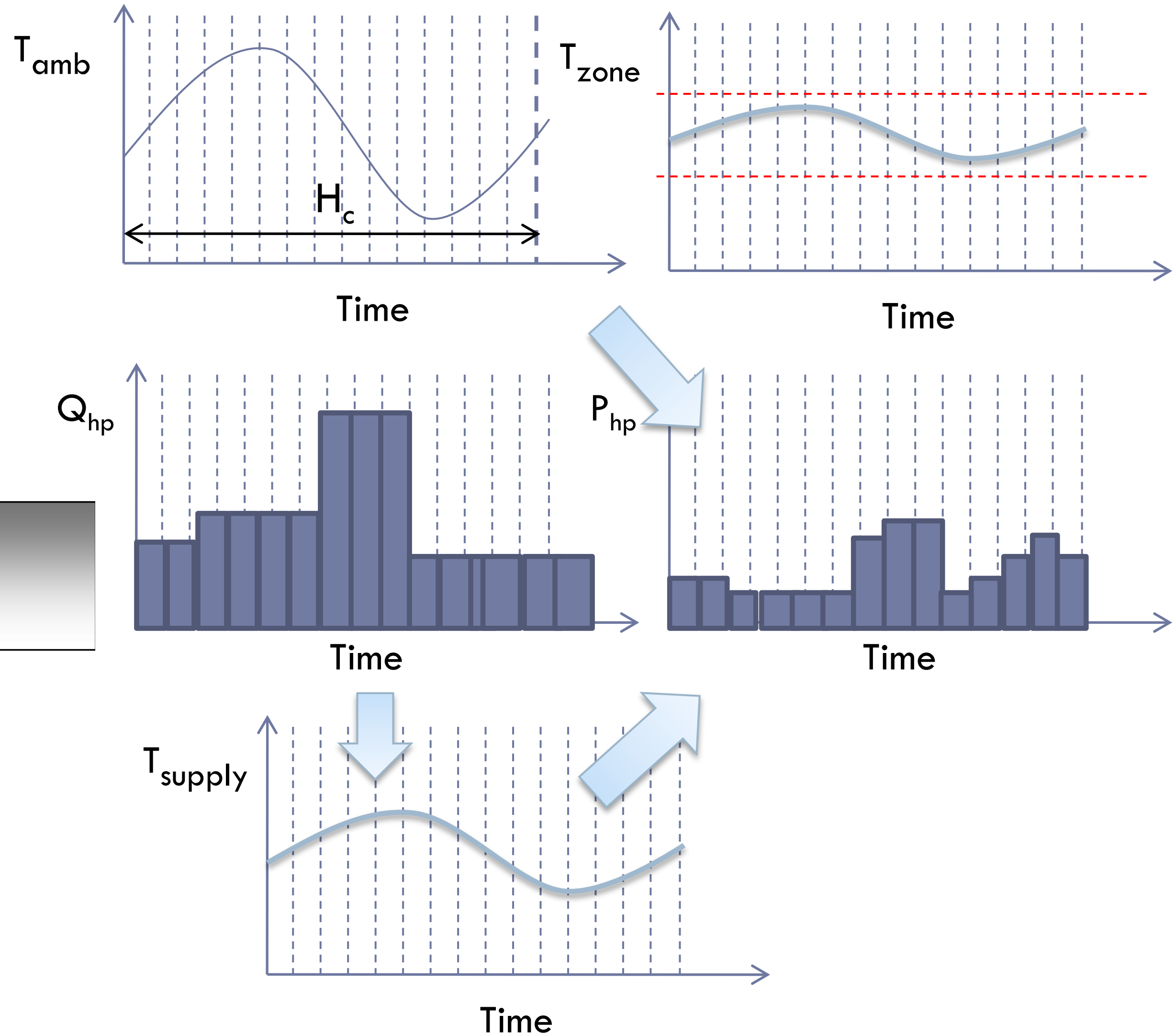


Building model

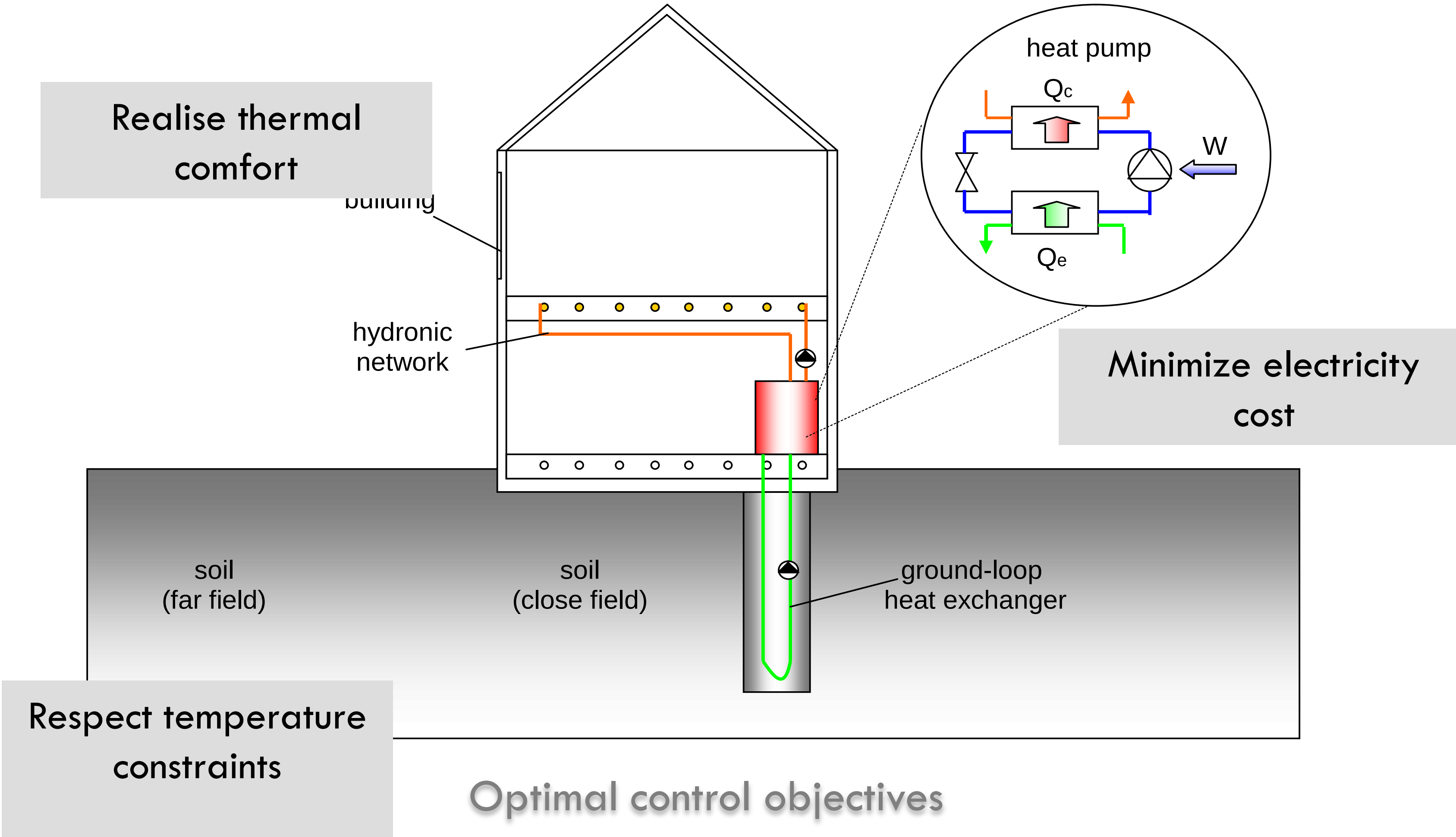
$T_{supply}, T_{zone} = f(T_{amb}, Q)$

Heat pump model

$COP = f(T_{amb}, T_{supply})$



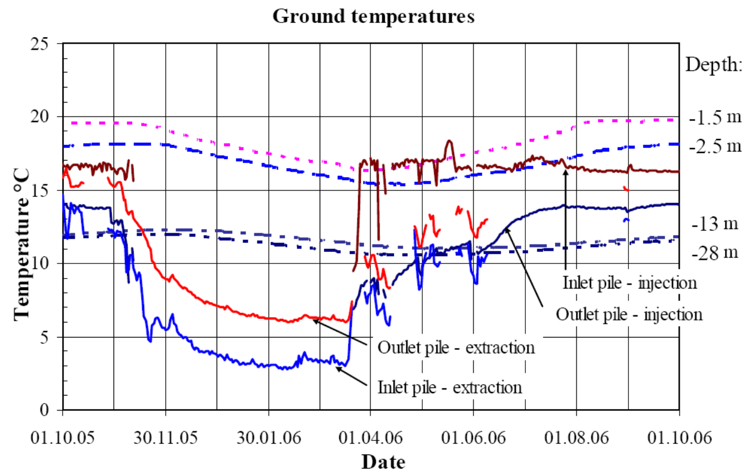
- Example 3: Ground coupled heat pump system



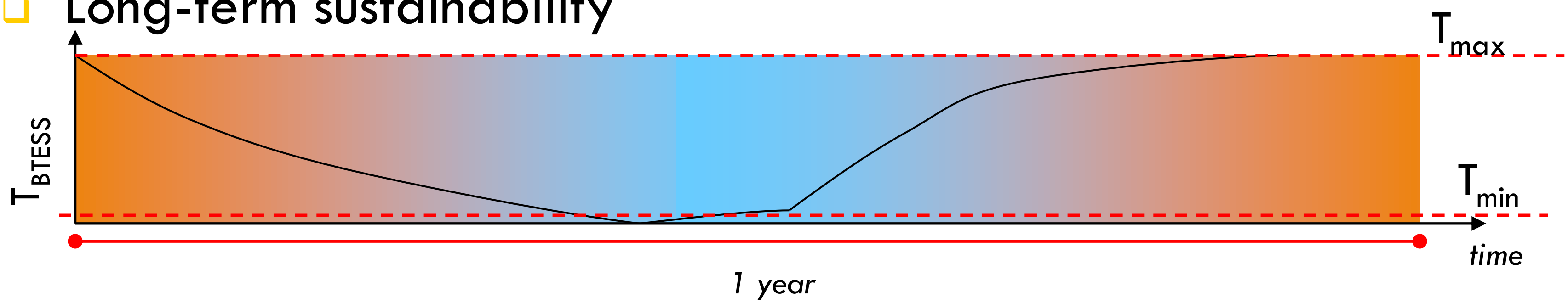
Optimal control objectives

Introduction

- Extra control objectives
  - Constraints on ground temperature

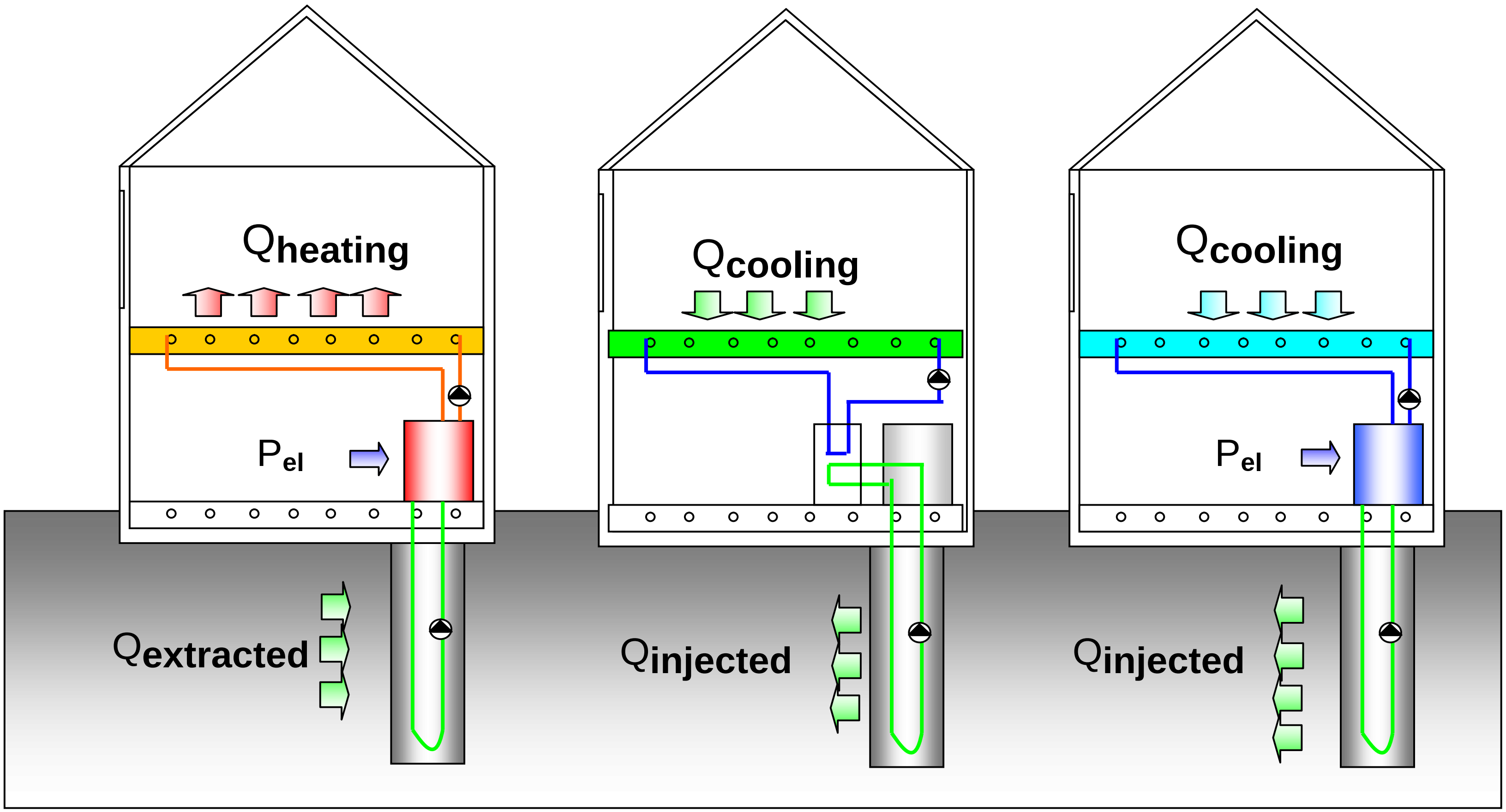


  - Long-term sustainability

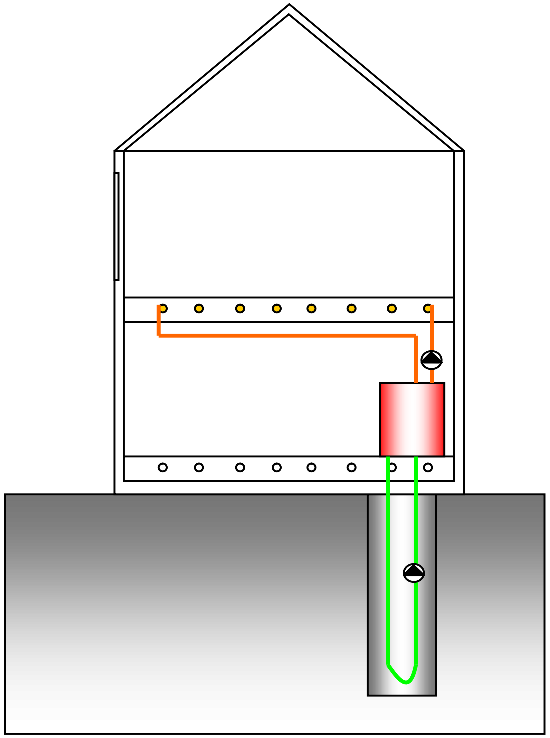


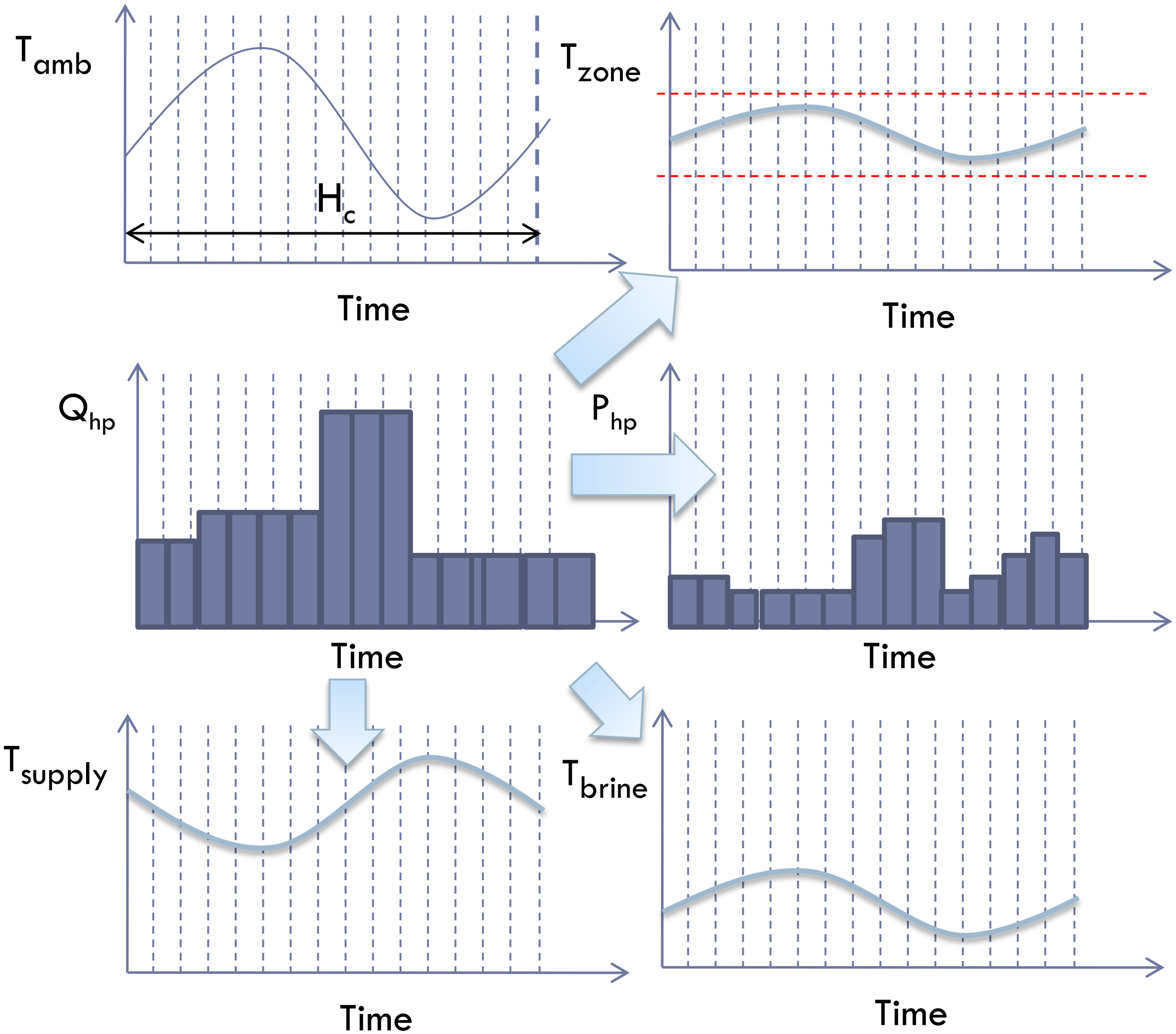
Source: Pahud and Hubbuck 2007

Introduction



Introduction



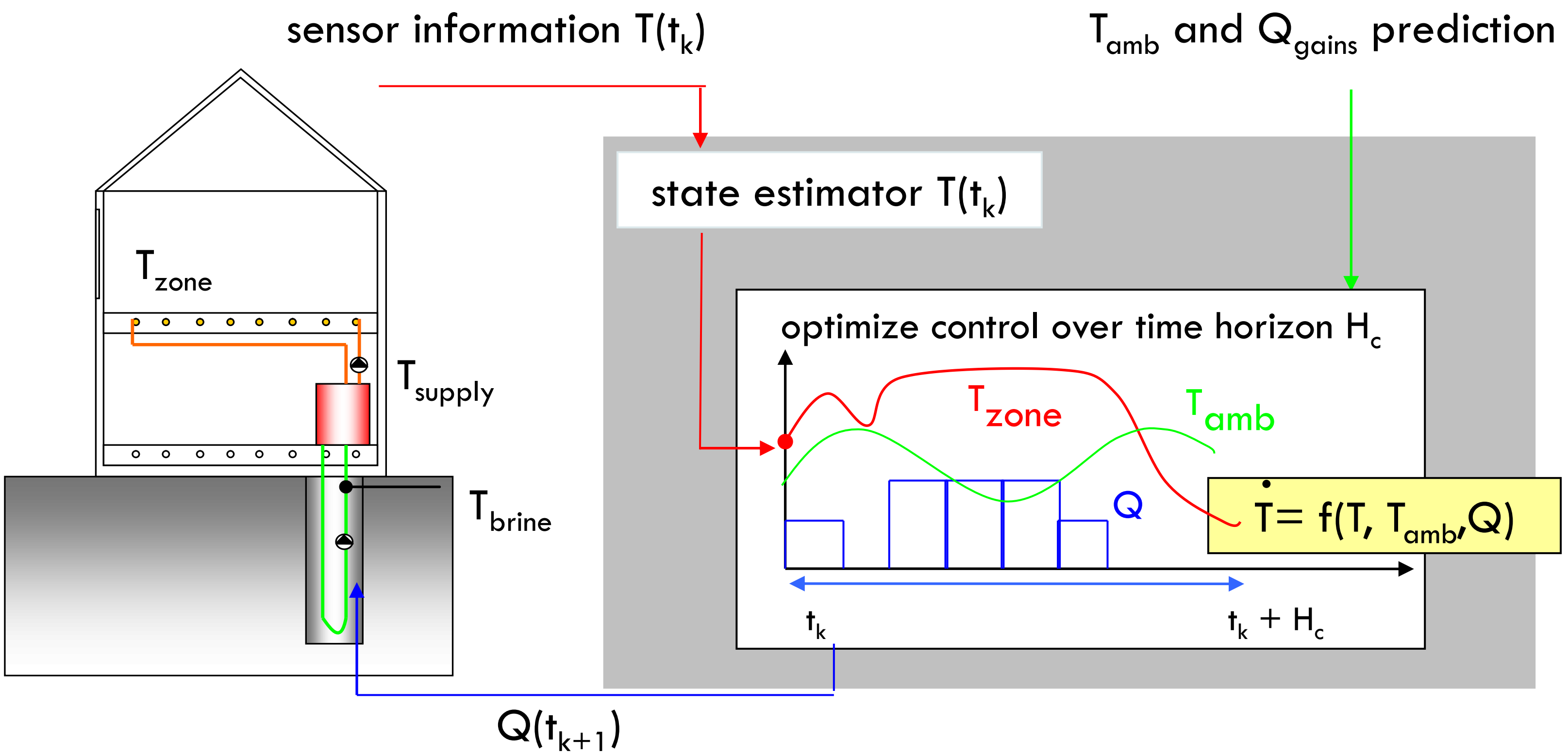


Building model
$T_{supply}, T_{zone} = f(T_{amb}, Q)$

Heat pump model
$COP = f(T_{brine}, T_{supply})$

Brine water model
$T_{brine} = f(T_{ground}, Q)$

- Example 3: Ground coupled HP system



Model Predictive Control scheme

Introduction

- Needed: Building model...



Source: www.groundMed.eu

Introduction

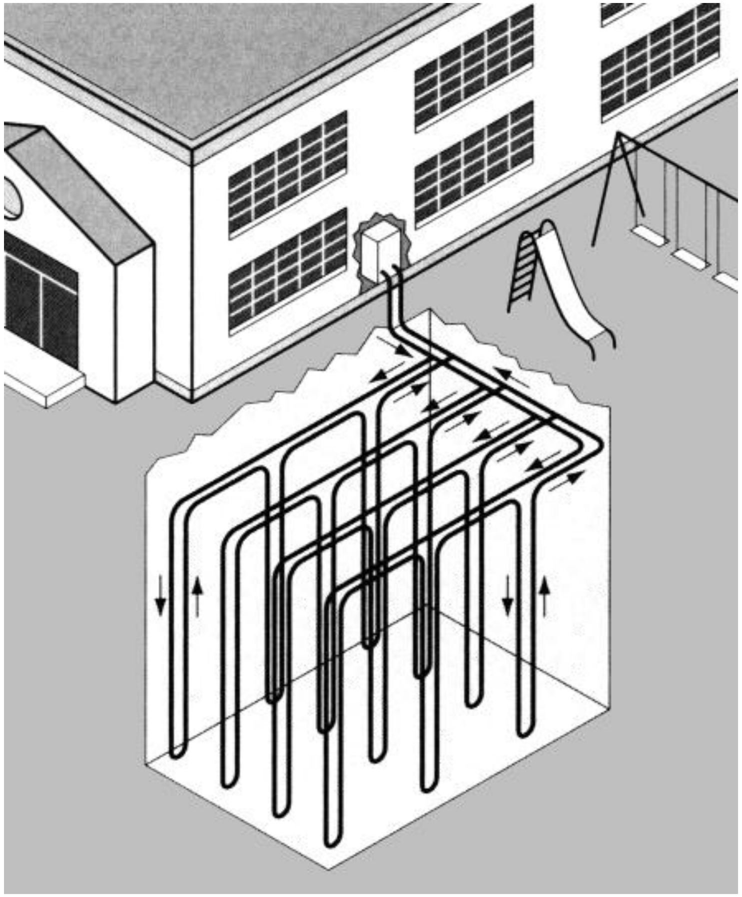
- Heating system model…

Source: Remeha

Introduction

- Borefield model…

Source: www.groundreach.eu



Source: [ww.geo4va.vt.edu](ww.geo4va.vt.edu)

Introduction: Summary

Why do we need a model for control?

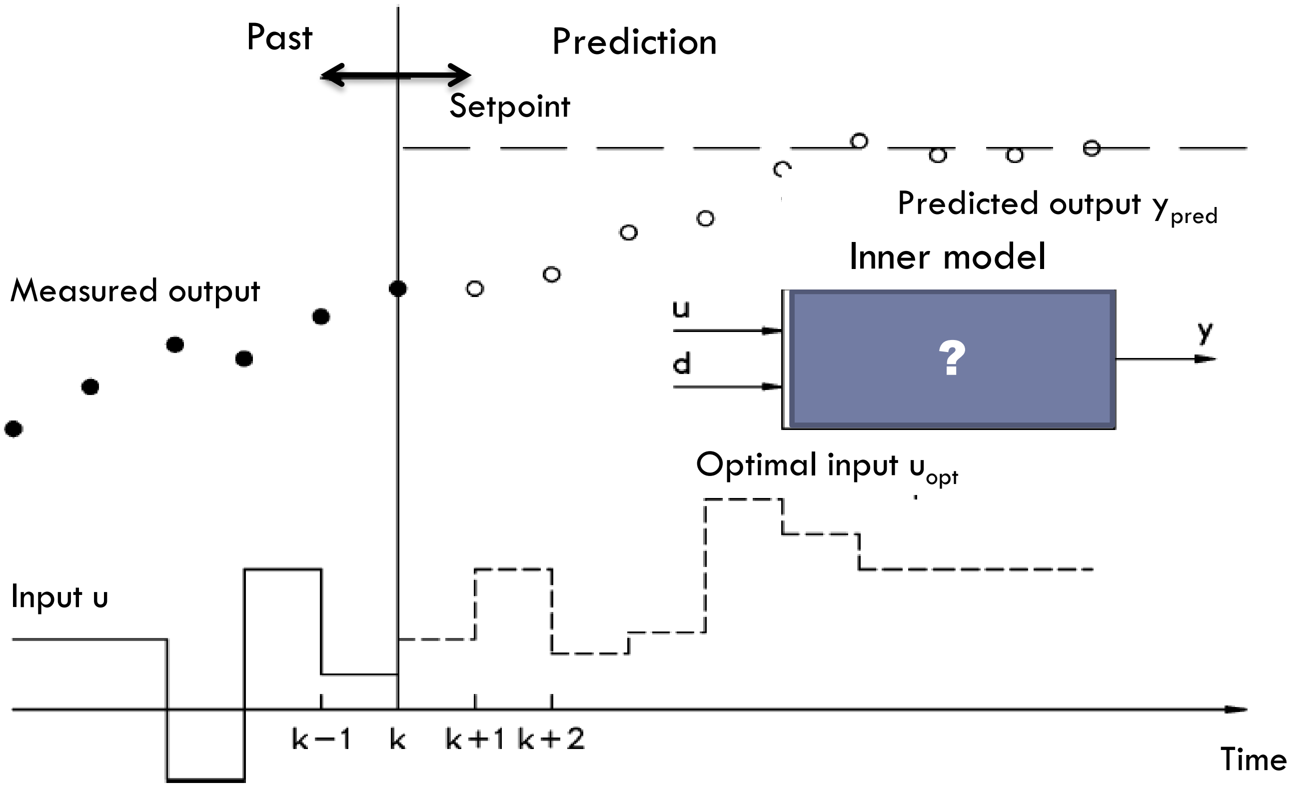
- Case studies: heating and cooling systems
  - Fast reacting systems, conventional control → static model
  - Slow reacting systems, model predictive control → dynamic model
  - Opportunities for optimization

Outline

- Introduction
- Framework of Model Predictive Control
- Development of control relevant model
- Applications in building control

MPC FRAMEWORK

- MPC in general



MPC framework

- Dynamic models used in MPC
  - Step response functions
  - Impulse response functions
  - Transfer functions
  - State space models
  - Ordinary Differential Equations
- Model complexity
  - Number of states
  - Nonlinearities
  - → "Not more complex than strictly needed"

MPC framework

- Physical versus blackbox models

Blackbox	Physical
No (less) prior knowledge required	Prior knowledge required
Selection blackbox model structures	Formulation system equations
Parameters no physical meaning	Parameters physical meaning
Only input-output relation	Physical insight in process
No extrapolation	Extrapolation

MPC framework

- Model types
  - Step response functions
  - Impulse response functions
  - Transfer functions
  - State space models
  - Ordinary Differential Equations

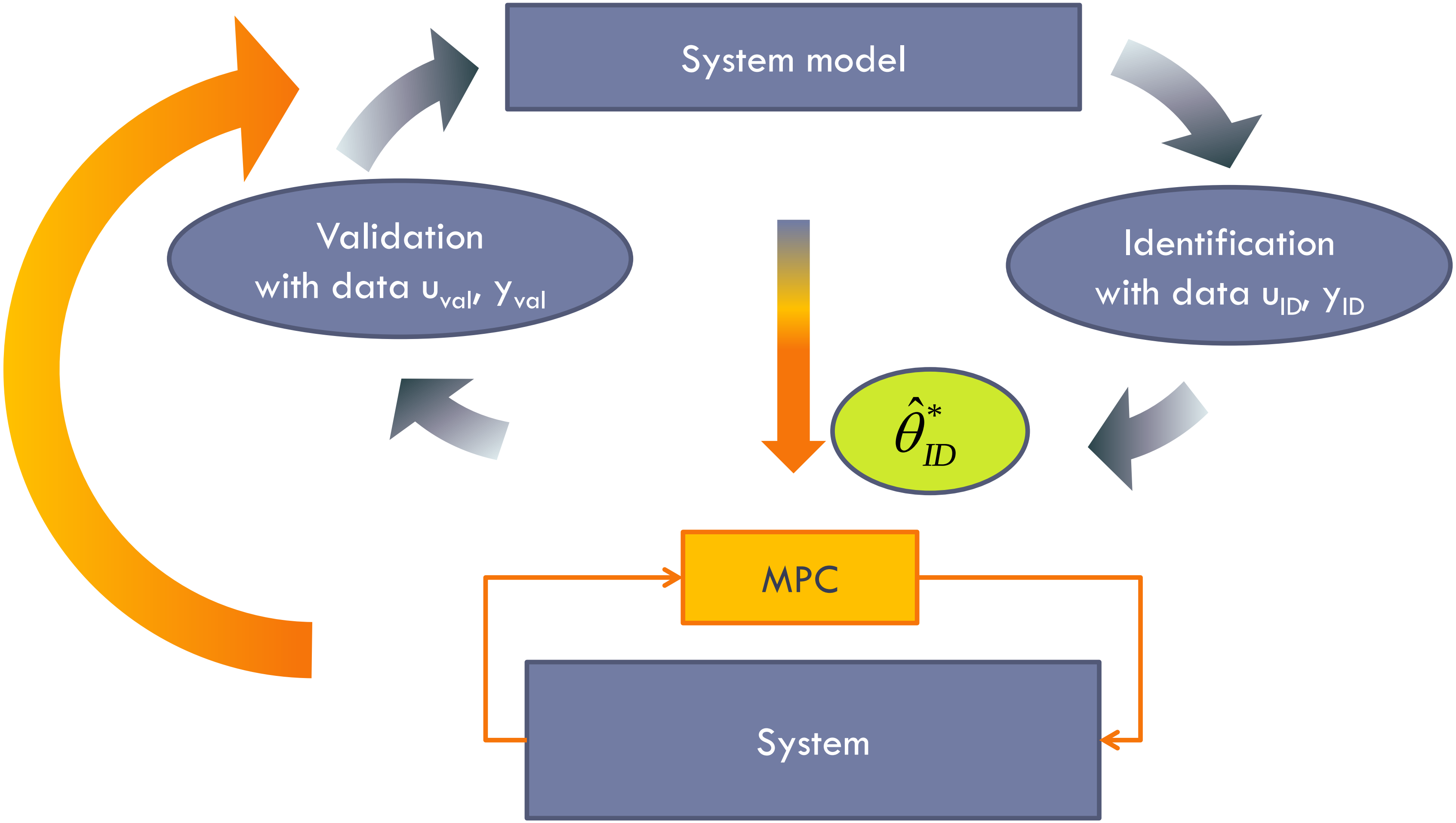
Physical models

Focus in this lecture

- Model complexity
  - Number of states
  - Nonlinearities
  - → "Not more complex than strictly needed"

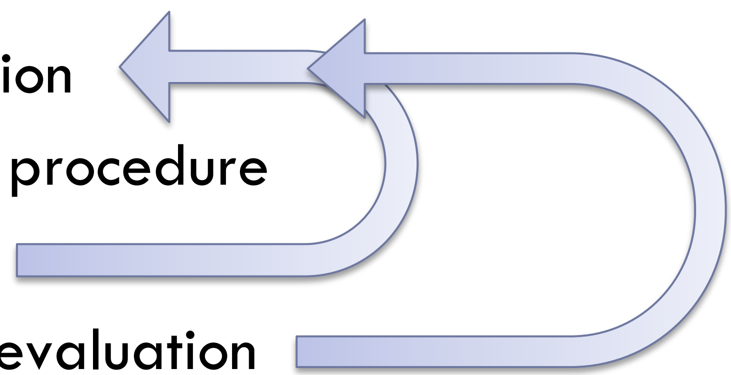
MPC framework

- Development of control relevant model



Outline

- Introduction
- MPC Framework
- Development of control relevant model
  - Physical modelling
  - Model structure selection
  - Parameter estimation procedure
  - Model validation
  - Control performance evaluation



- Applications in building control

References

- Maciejowski, J. M., Ed. (2002). Predictive control: with constraints. Pearson Education. Great Britain.

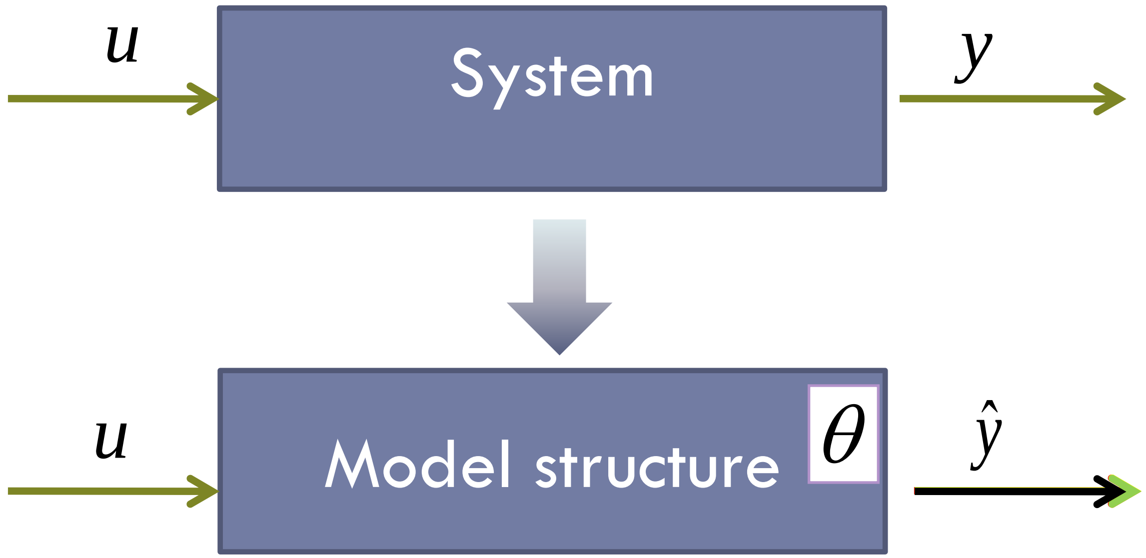
DEVELOPMENT OF CONTROL RELEVANT MODEL

Development of a control relevant model

- Step 1: Define system boundaries
- Step 2: Select model structure
- Step 3: Identify model parameters
- Step 4: Validate model
  - If bad fit: Try better identification data
  - If still bad fit: Try other model structure
- Step 5: Evaluate control performance

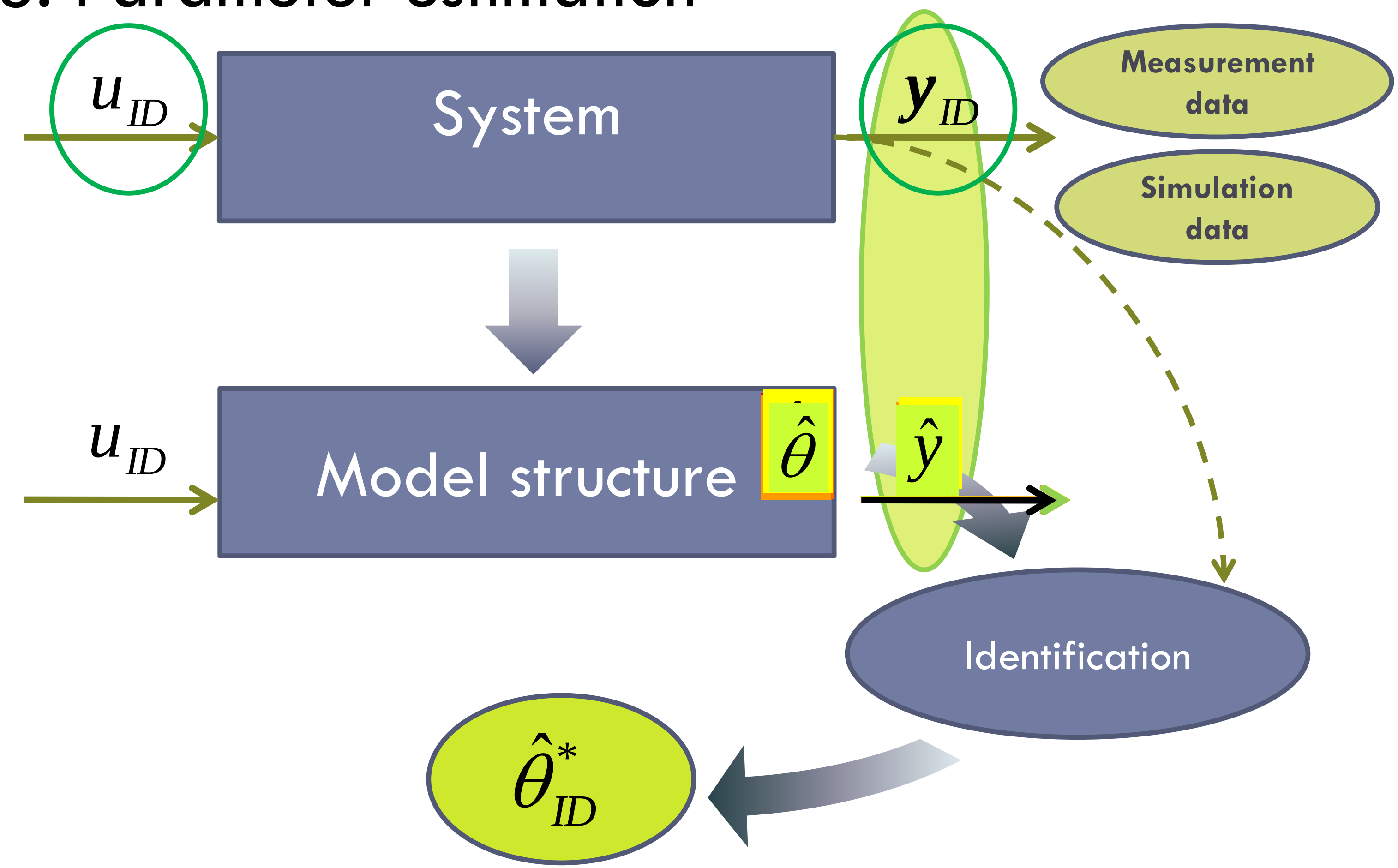
Development of a control relevant model

- Steps 1 & 2: Model structure selection



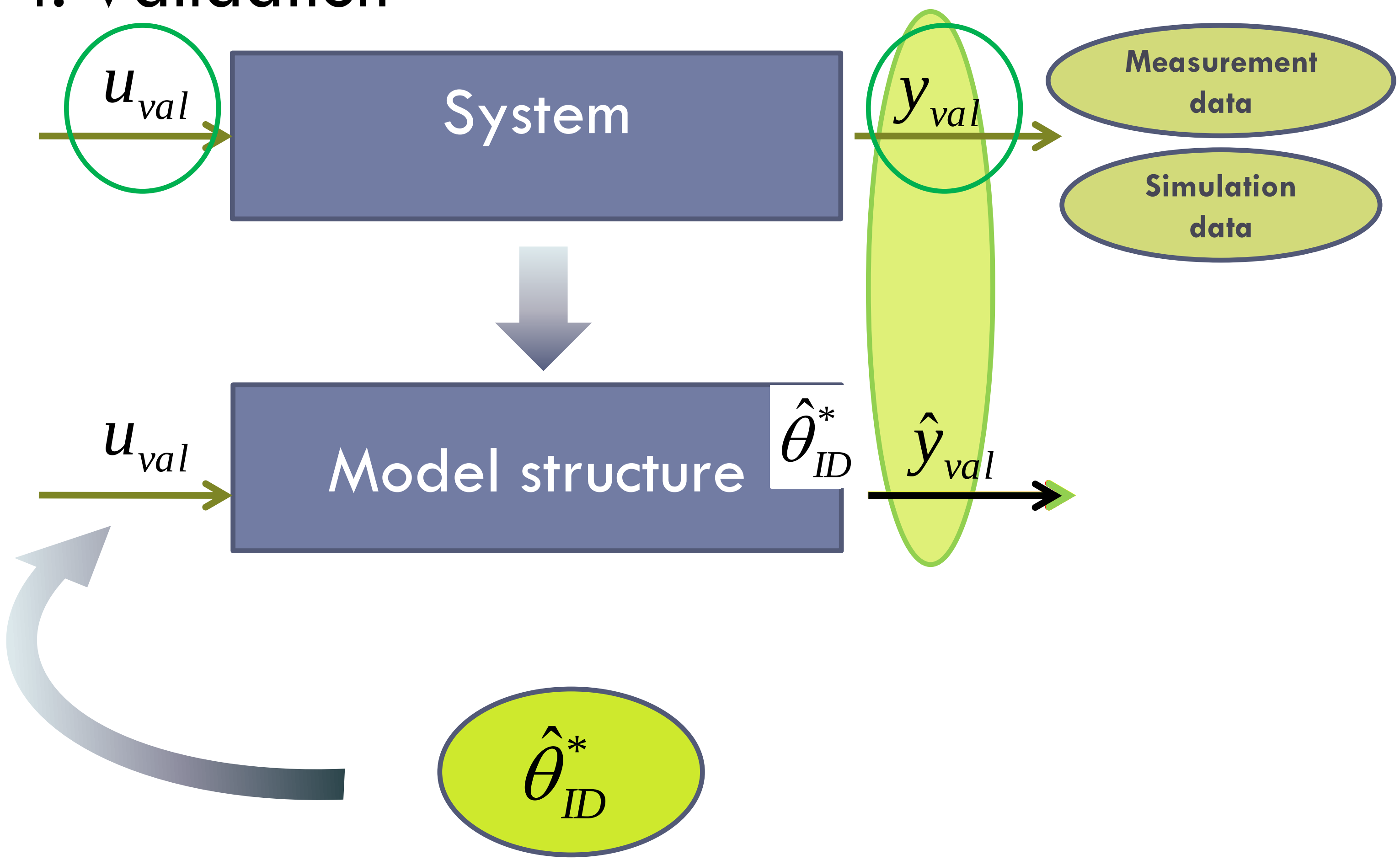
Development of a control relevant model

- Step 3: Parameter estimation



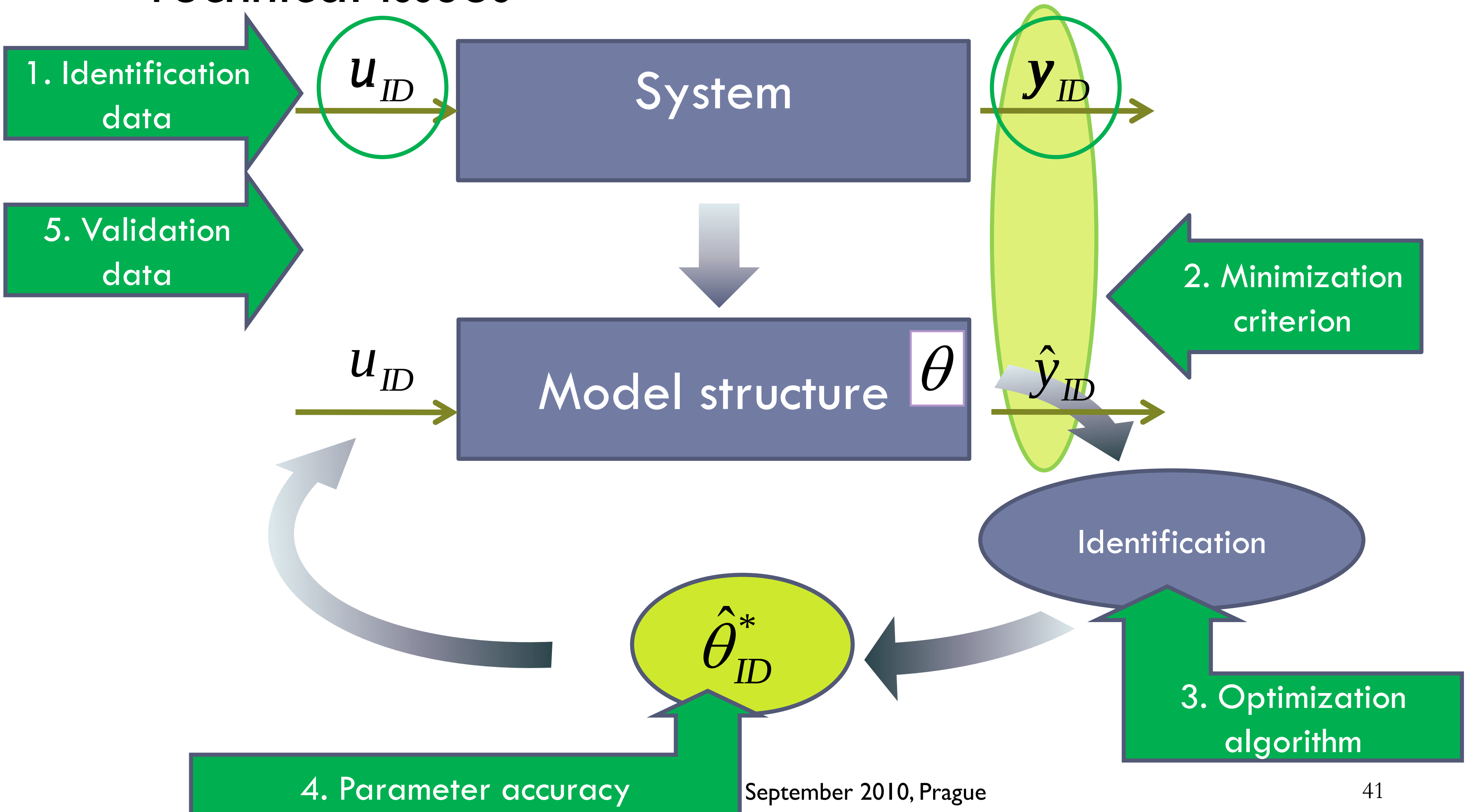
Development of a control relevant model

- Step 4: Validation



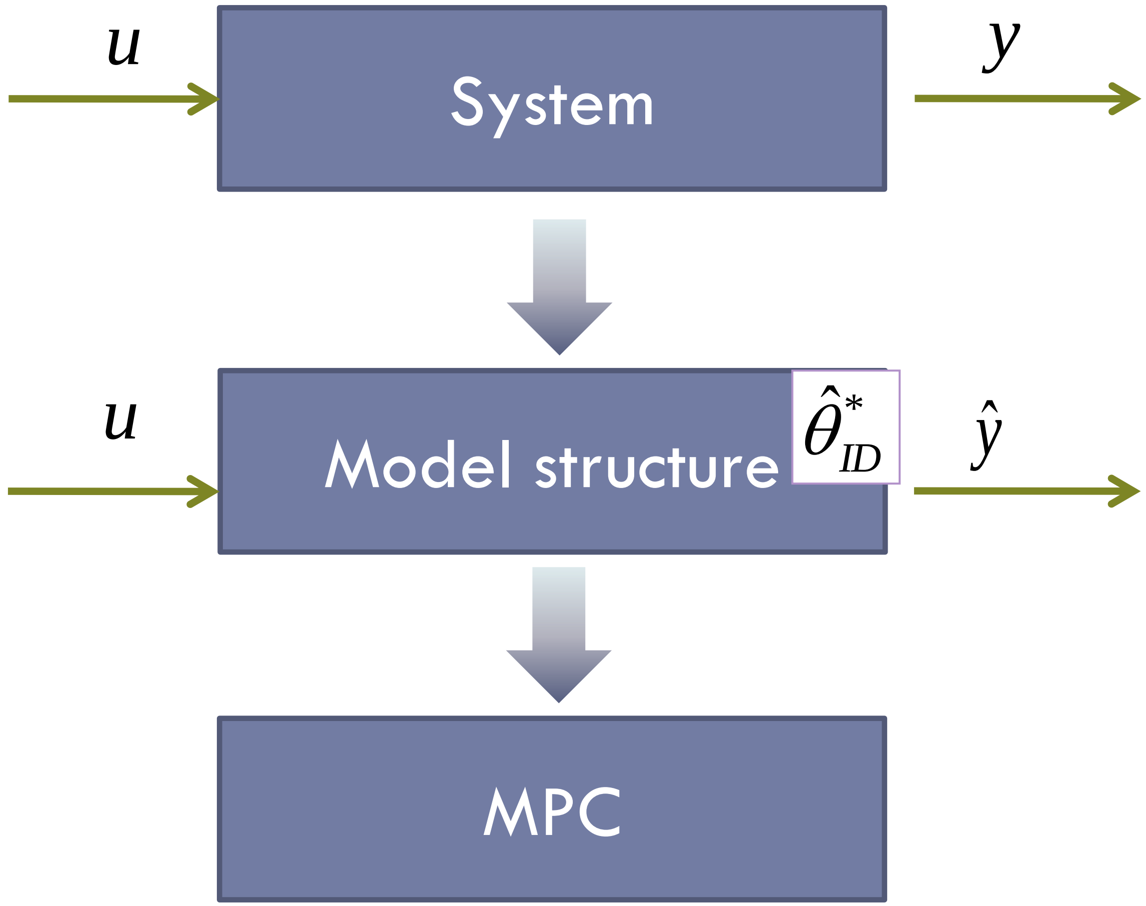
Development of a control relevant model

- Technical issues



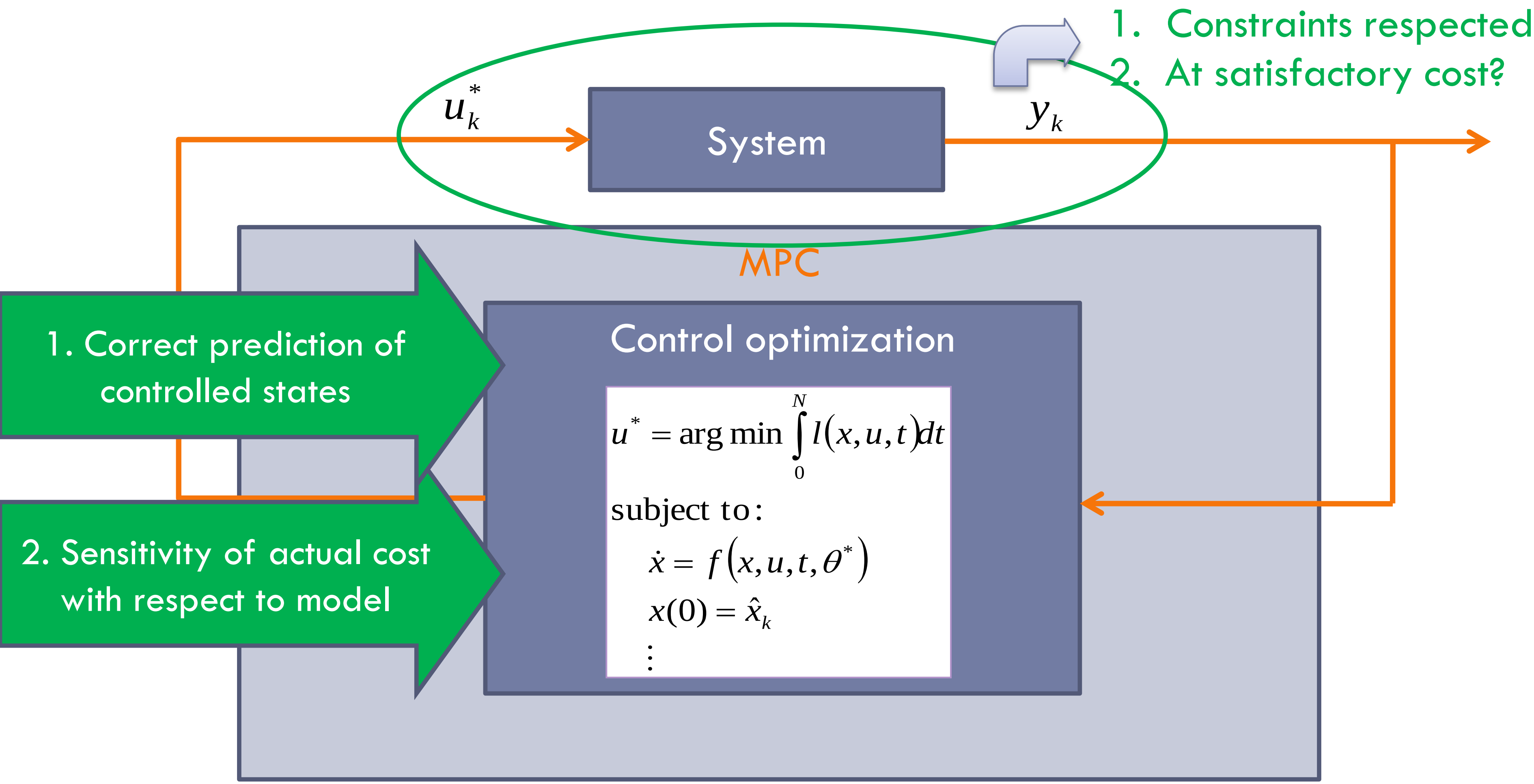
Development of a control relevant model

- Step 5: Control performance evaluation



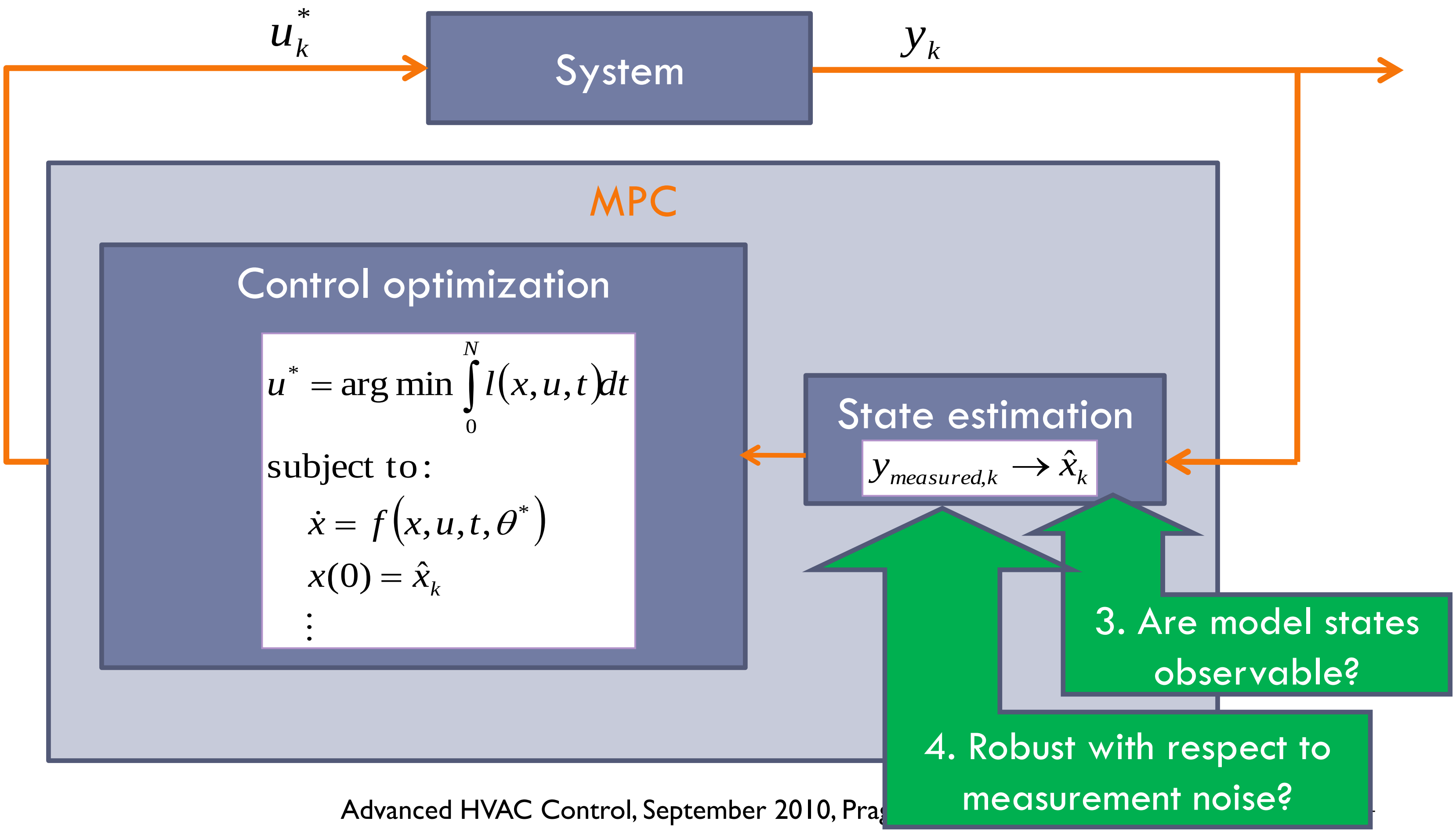
Development of a control relevant model

- Step 5: Is the exptected control performance achieved?



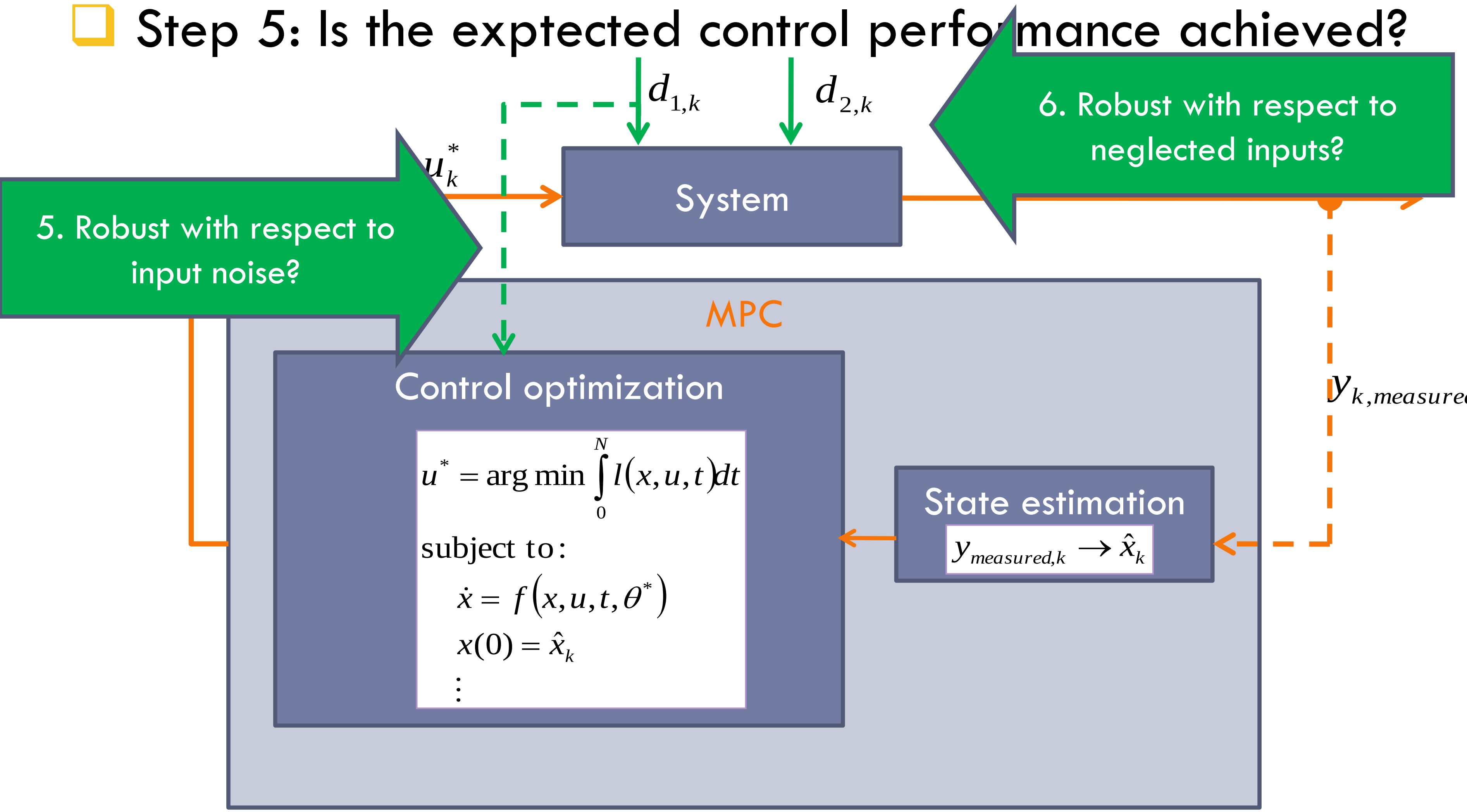
Development of a control relevant model

- Step 5: Is the exptected control performance achieved?



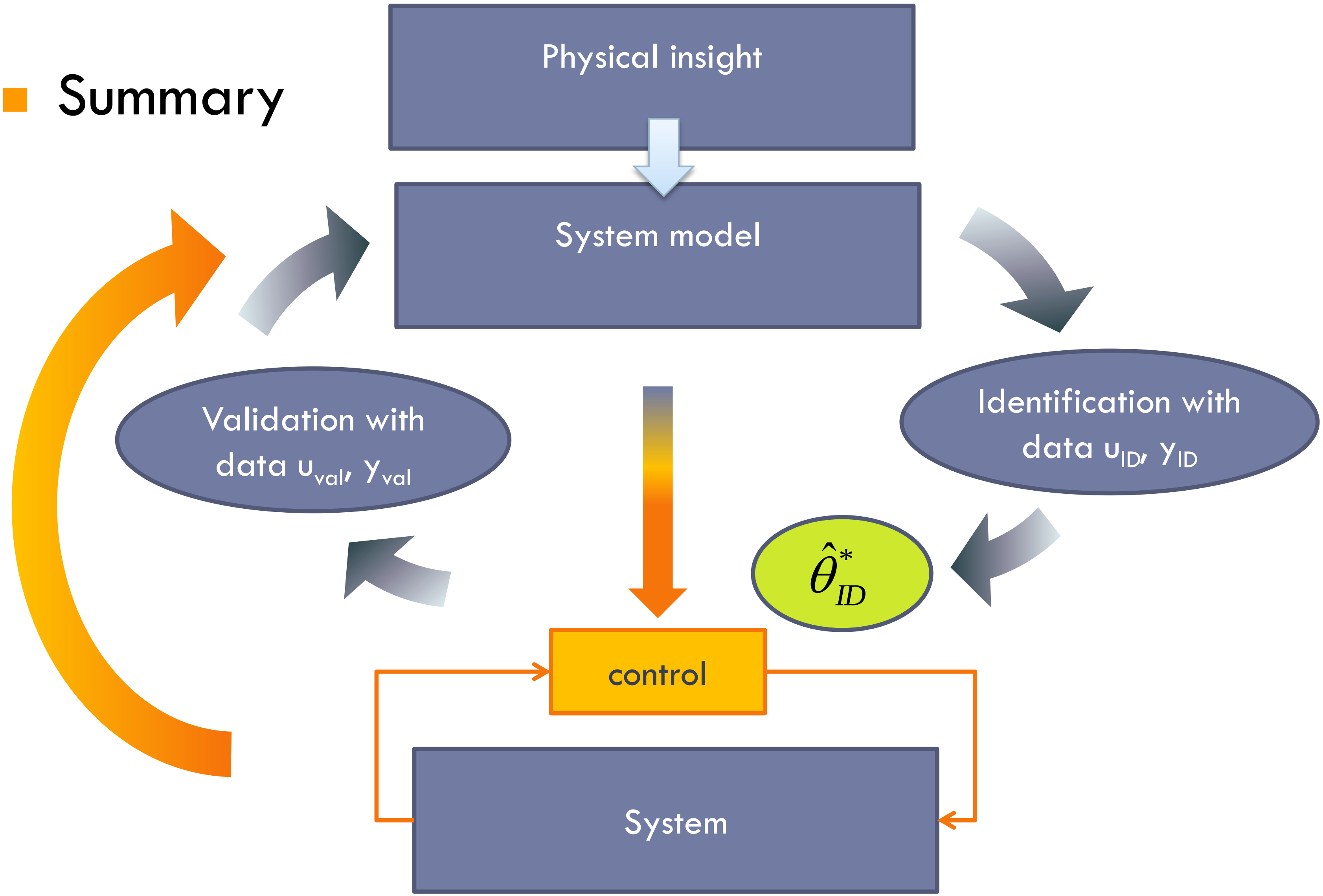
Development of a control relevant model

- Step 5: Is the exptected control performance achieved?



Development of control relevant model

- Summary



Outline

- Introduction
- Framework of Model Predictive Control
- Development of control relevant model
- Applications in building control

Applications in building control

- Applications in building control
  - Heating curve control
  - MPC for heavy-weight solar building
  - MPC for heat pump system with floor heating
  - MPC for ground coupled heat pump system
  - MPC for multizone building





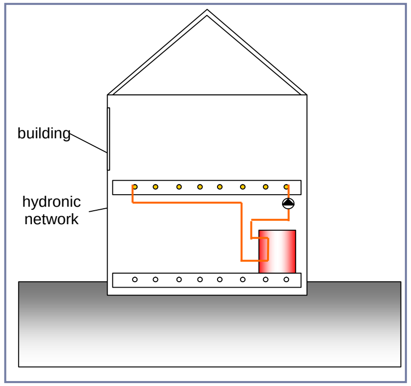
Heating curve example

- 1. Control objective

$$Q_{heater} \rightarrow \boxed{T_{water}} \rightarrow \boxed{T_{zone}} \rightarrow T_{amb}$$

control variable (under T_{water})

controlled variable (under T_{zone})

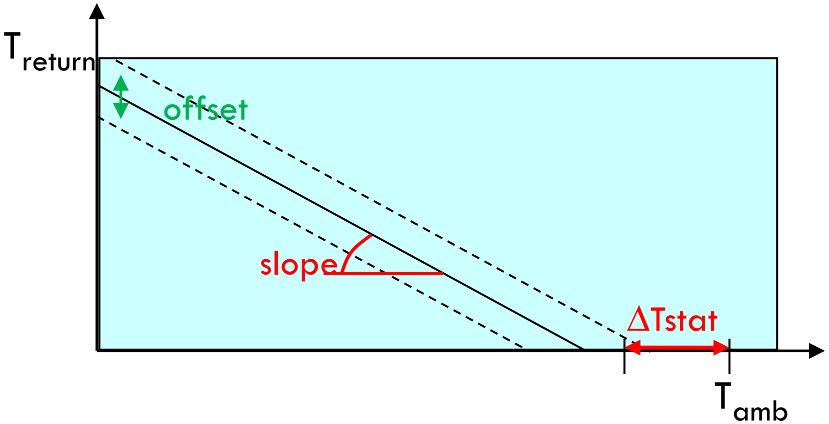


- 2. Physical model

$$C_z \frac{dT_{zone}}{dt} = \dot{Q}_{rad} + \dot{Q}_{gains} - \dot{Q}_{loss}$$

$$\rightarrow \dot{Q}_{rad} = UA_{rad}\left(\overline{T}_{water} - T_{zone}\right)$$

$$\rightarrow \dot{Q}_{loss} = UA_{building}\left(T_{zone} - T_{amb}\right)$$



- Assume steady-state

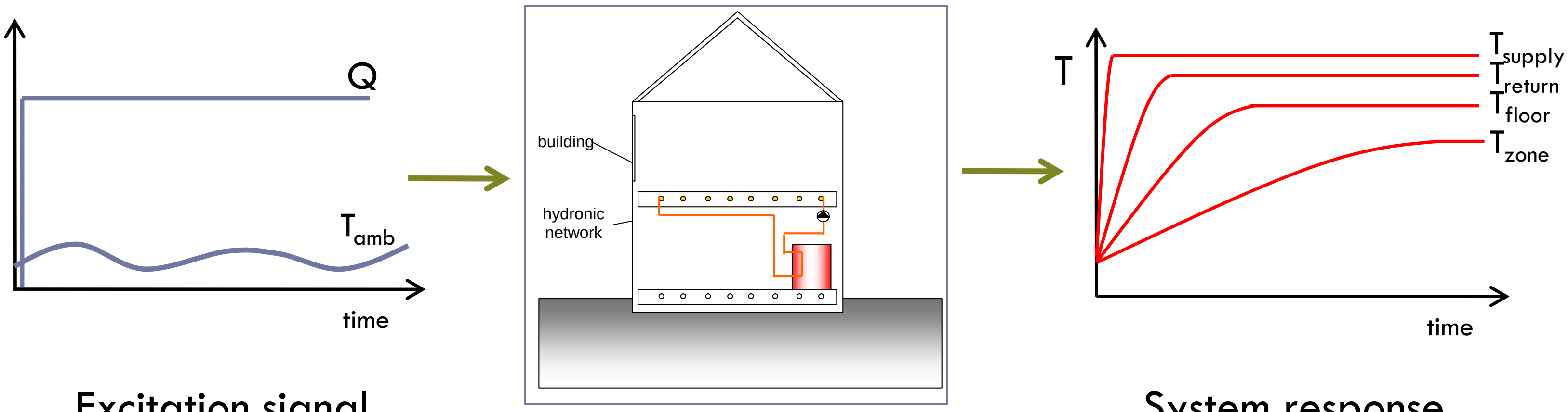
$$\dot{Q}_{rad} + \dot{Q}_{gains} - \dot{Q}_{loss} = 0 \quad \Rightarrow \quad \overline{T}_w = T_z + \boxed{\frac{UA_{building}}{UA_{rad}}}\left(T_z - \left(T_{amb} + \boxed{\Delta T_{stat}}\right)\right)$$

Heating curve example

- 3. Model structure
  - linear, algebraic equation
  - 2 parameters

$$\bar{T}_w = T_z + \boxed{\frac{UA_{building}}{UA_{rad}}}\left(T_z - \left(T_{amb} + \boxed{\Delta T_{stat}}\right)\right)$$

- 4. Parameter identification
  - Identification data



Excitation signal

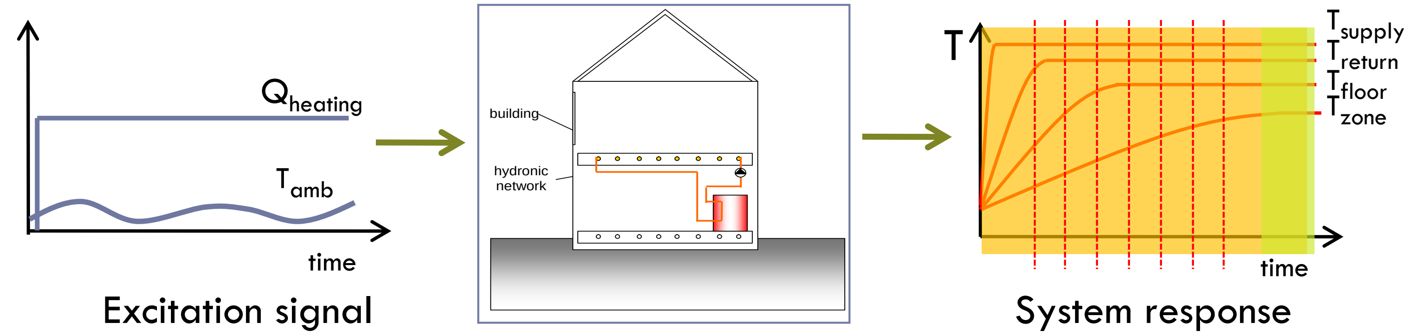
u_{ID}

System response

y_{ID}

Heating curve example

- **4. Parameter estimation**
  - Identification data



Excitation signal u_{ID}

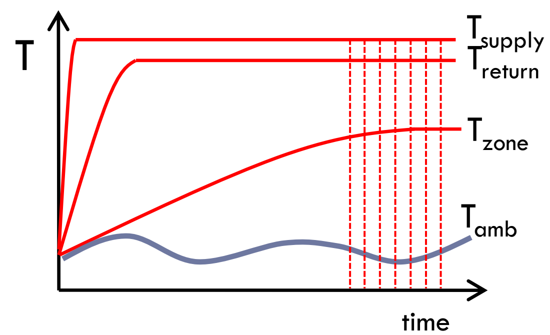
System response y_{ID}

  - Measurement setup
    - Selection:
      - Measured variables $\{ T_{supply}, T_{return}, T_{floor}, T_{zone}, T_{amb}, Q \}$
      - Measurement period
      - Sampling frequency
    - Be aware of measurement noise!

Heating curve example

- **4. Parameter estimation**
  - Model structure

$$\bar{T}_w = T_z + \boxed{\frac{UA_{building}}{UA_{rad}}}\left(T_z - \left(T_{amb} + \boxed{\Delta T_{stat}}\right)\right)$$



  - Identification data

→ m measurements T_{water}, T_{zone}, T_{amb}

  - Linear regression

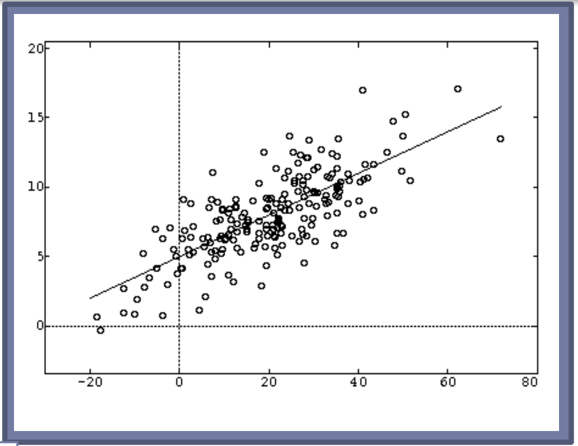
$$\begin{bmatrix} \bar{T}_{water}(0) - T_{zone}(0) \\ \vdots \\ \bar{T}_{water}(m) - T_{zone}(m) \end{bmatrix} = \begin{bmatrix} T_{zone}(0) - T_{amb}(0) & 1 \\ \vdots & \vdots \\ T_{zone}(m) - T_{amb}(m) & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

with
$$\begin{cases} \theta_1 = \dfrac{UA_{building}}{UA_{rad}} \\ \theta_2 = \dfrac{UA_{building}}{UA_{rad}} \Delta T_{stat} \end{cases}$$

$$\varphi = X\theta$$

$$\theta^* = \left(\varphi'\varphi\right)^{-1}\varphi' X$$

Intermezzo



- Least squares
  - Optimal estimator

$$e(t) = \hat{y}(t,\theta) - y(t)$$

$$E\{e(t)\} = 0$$
$$E\{e(t)' e(t)\} = \Lambda$$

$$\boxed{\theta^* = \arg\min E\left\{\frac{e(t)' e(t)}{\Lambda}\right\}}$$

  - In case only white noise at output

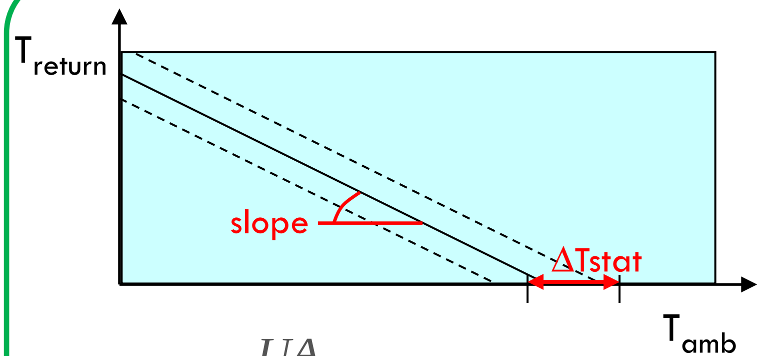
$$E\{e(t)\} = 0$$
$$E\{e(t)' e(t)\} = \sigma^2 I$$

$$\boxed{\theta^* = \arg\min E\left\{\sum_{n_y} \frac{e_i(t)' e_i(t)}{\sigma_i^2}\right\}}$$

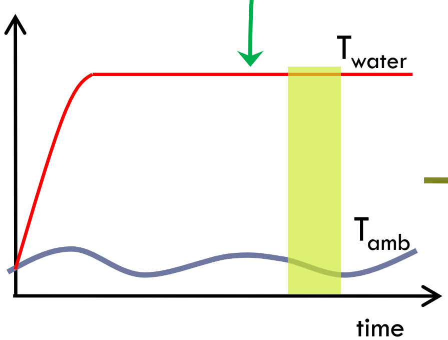
  - ... If also linear in parameters

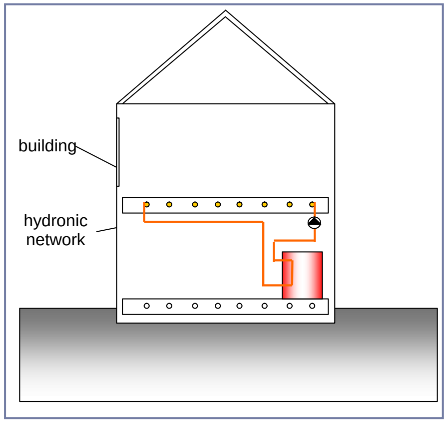
$$\boxed{\theta^* = (\varphi' \varphi)^{-1} \varphi' X}$$

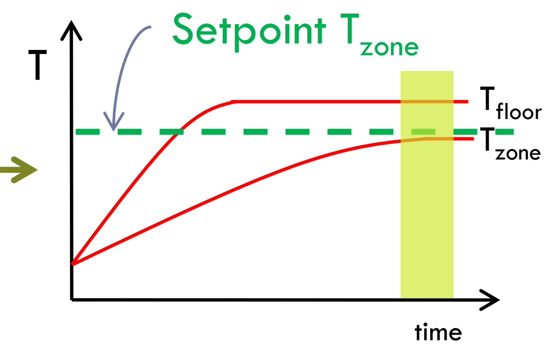
- 5. Model validation
  - "Expected steady state behavior?"



$$\bar{T}_w = T_z + \frac{UA_{building}}{UA_{rad}}\left(T_z - \left(T_{amb} + \Delta T_{stat}\right)\right)$$







Validation input data

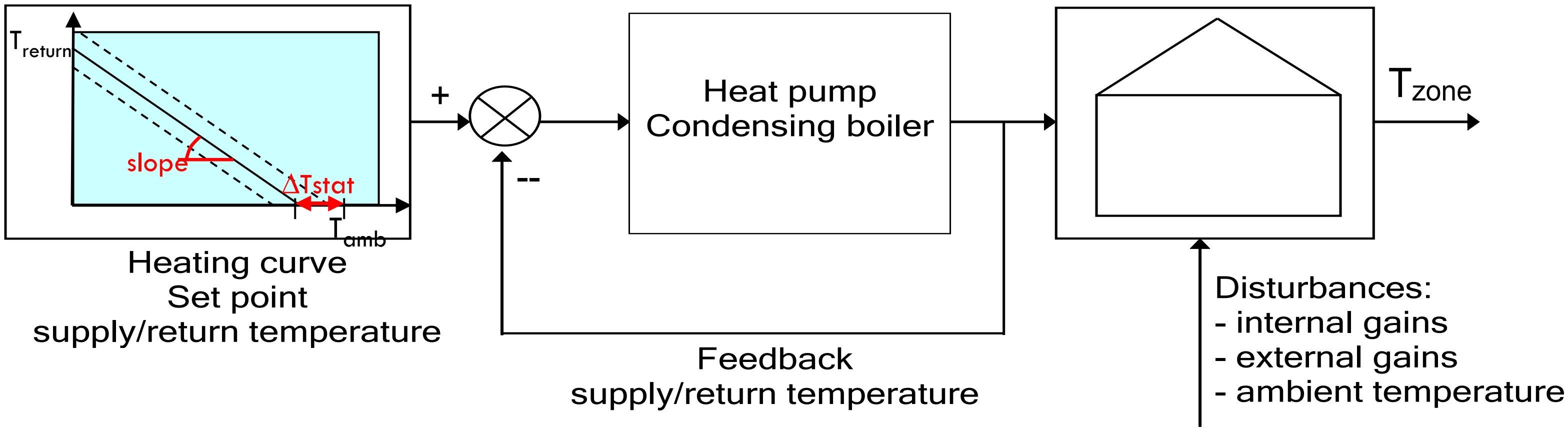
u_{val}

System response

y_{val}

Heating curve example

- 6. Evaluate control performance



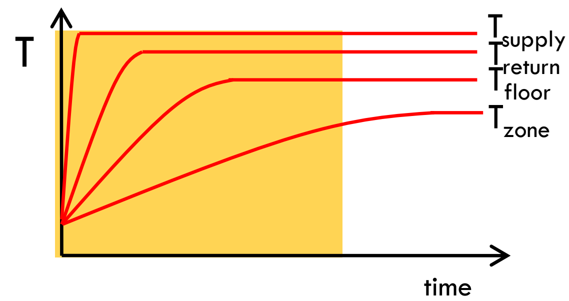
  - If unsatisfactory… Maybe static building model not appropriate?

Heating curve example

- Discussion: static versus dynamic control model
  - From m steady state measurements T_{water}, T_{zone}, T_{amb}

    → static model
  - If also *transient data* measured:

    → dynamic model

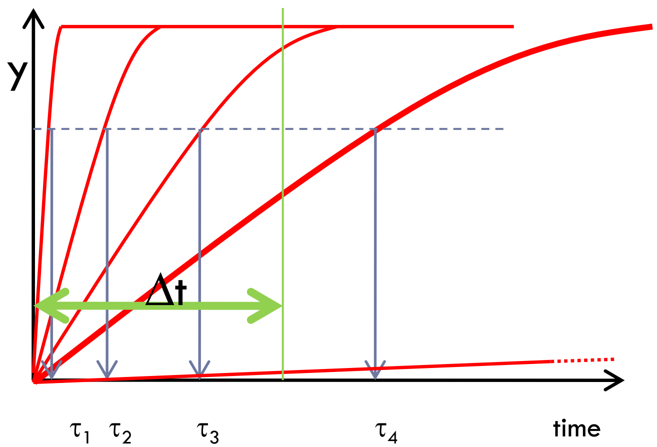


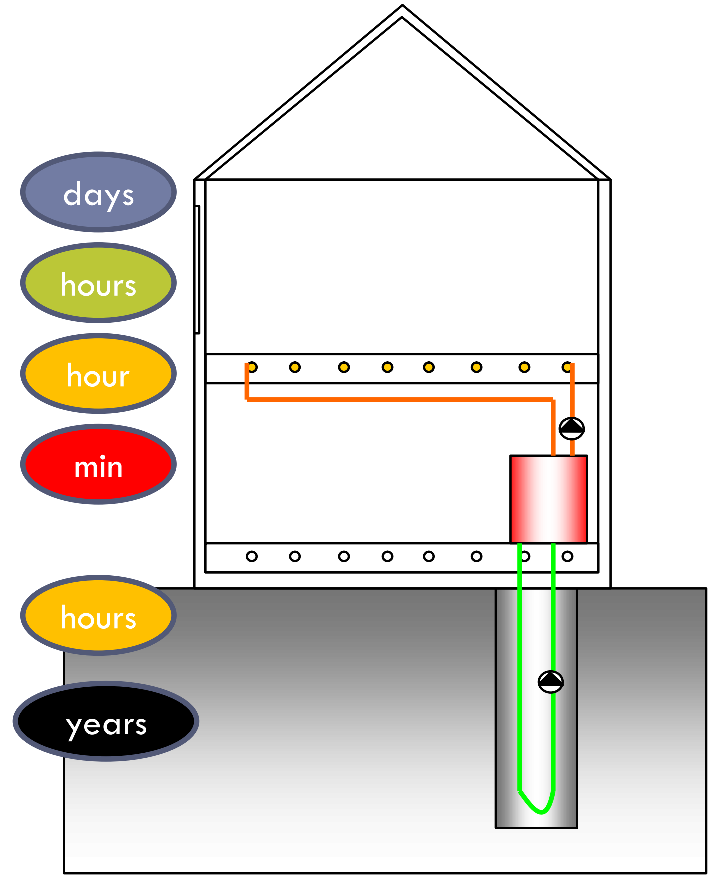
$$\frac{dT_{zone}}{dt} = \boxed{\frac{UA_{rad}}{C_{zone}}}\left(\overline{T}_{water} - T_{zone}\right) + \boxed{\frac{UA_{building}}{C_{zone}}}\left(T_{zone} - \left(T_{amb} + \Delta T_{stat}\right)\right)$$

$$\begin{bmatrix} T_{zone}(1) - T_{zone}(0) \\ \vdots \\ T_{zone}(m) - T_{zone}(m-1) \end{bmatrix} = \Delta t_s \begin{bmatrix} \overline{T}_{water}(0) - T_{zone}(0) & T_{zone}(0) - \left(T_{amb}(0) - \Delta T_{stat}\right) \\ \vdots & \vdots \\ \overline{T}_{water}(m) - T_{zone}(m) & T_{zone}(m-1) - \left(T_{amb}(m-1) - \Delta T_{stat}\right) \end{bmatrix} \begin{bmatrix} \theta_4 \\ \theta_5 \end{bmatrix}$$

Heating curve example

- Discussion: static versus dynamic control model





- Implications
  - Too quick: static model
  - Too slow: constant value
  - In between: dynamic model

Outline

- Introduction
- Framework of Model Predictive Control
- Development of control relevant model
- Applications in building control

Applications in building control

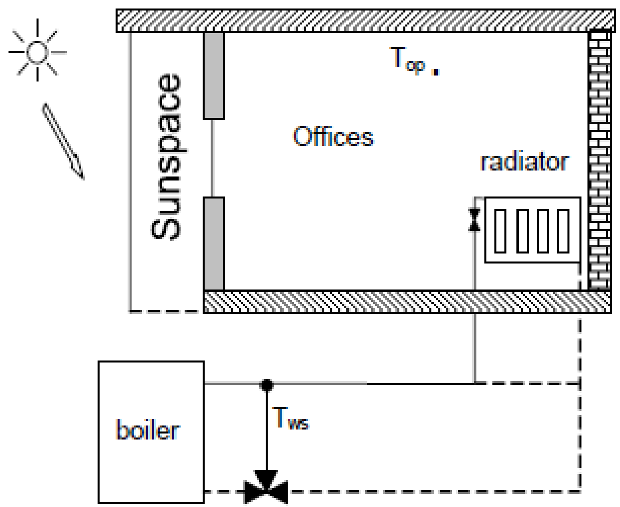
- Applications in building control
  - Heating curve control
  - MPC for heavy-weight solar building
  - MPC for heat pump system with floor heating
  - MPC for ground coupled heat pump system
  - MPC for multizone building





Solar building example

- 1. Control objective
  - Thermal comfort in heavy-weight building with radiators



Single zone with radiator

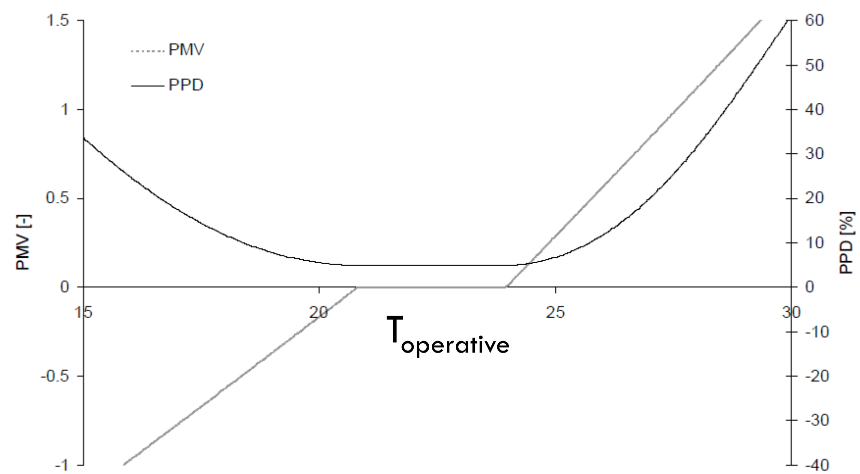


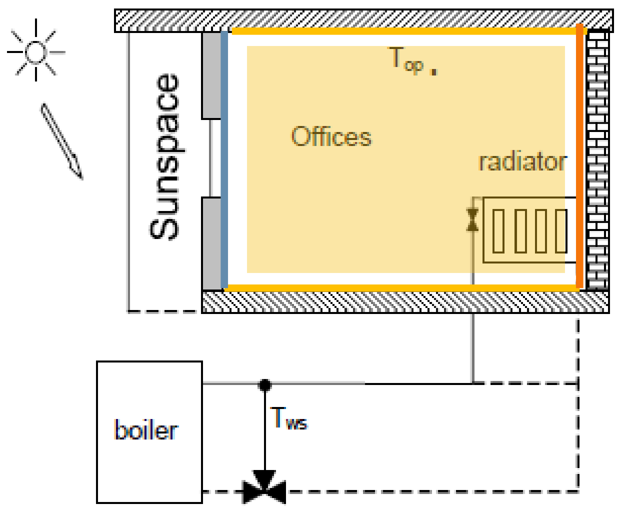
Figure 3-3: PPD and PMV with variable clothing

Objective: thermal comfort

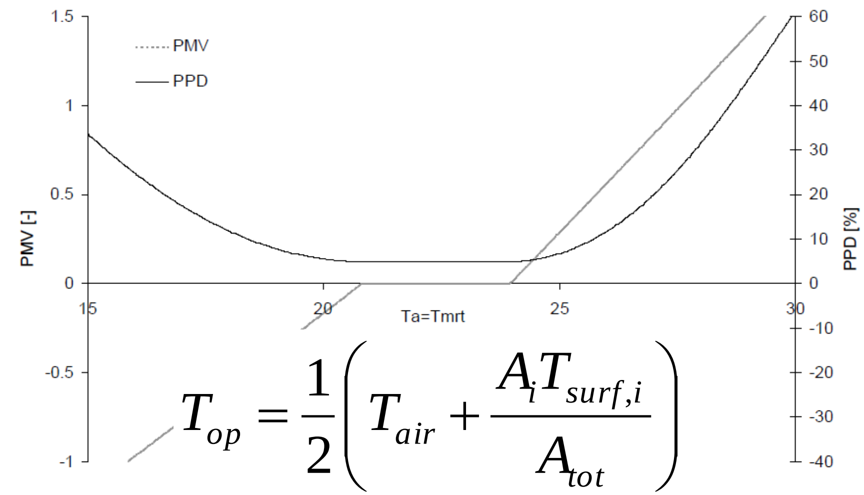
Control variables – Controlled variables

Source: Kummert 2001

- 1. Control objective
  - Control of thermal comfort in single-zone with radiators



Single zone with radiator



$$T_{op} = \frac{1}{2}\left(T_{air} + \frac{A_i T_{surf,i}}{A_{tot}}\right)$$

Figure 3-3: PPD and PMV with variable clothing

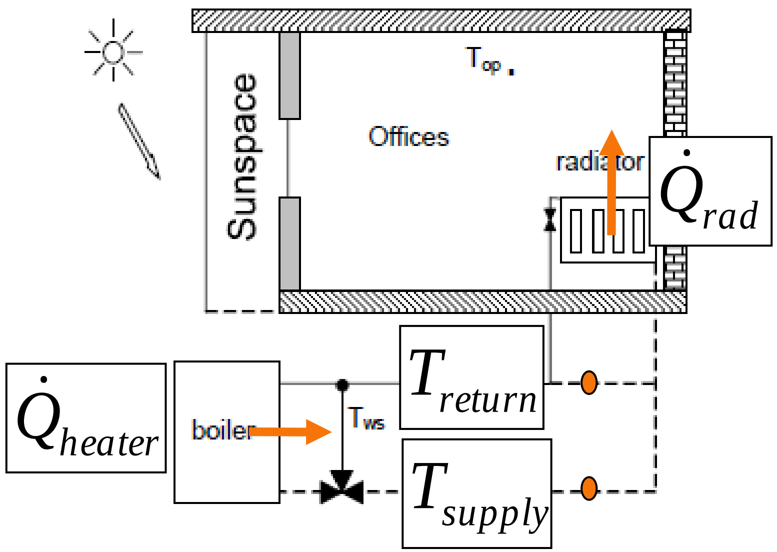
Objective: thermal comfort

Control variable – Controlled variables

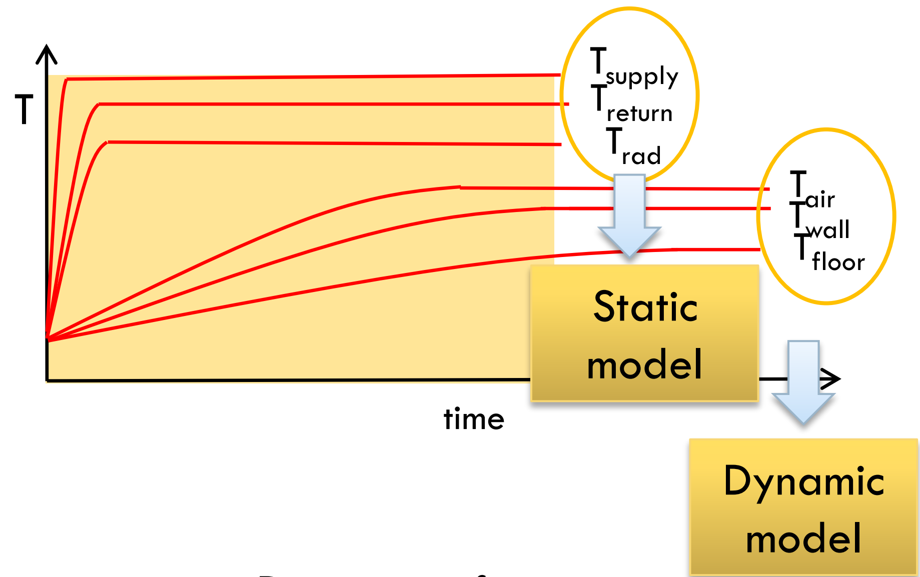
Source: Kummert 2001

Solar building example

- 1. Control objective
  - Control of thermal comfort in single-zone with radiators



Single zone with radiator

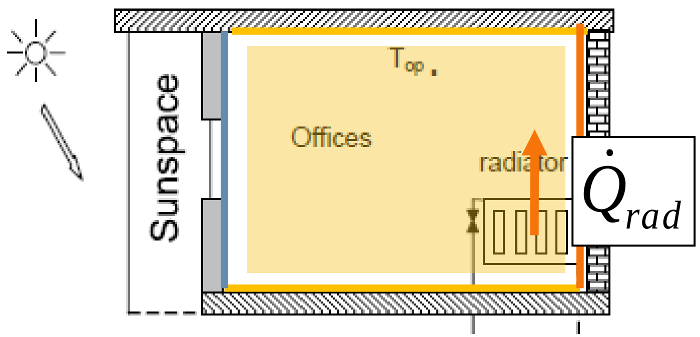


Dynamics of interest

Control variables – Controlled variables

Solar building example

- 1. Control objective
  - Control of thermal comfort in single-zone with radiators



$$T_{op} = \frac{1}{2}\left(T_{air} + \frac{A_i T_{surf,i}}{A_{tot}}\right)$$

$\dot{Q}_{rad}$ → **Building model** → T_{air}, $T_{wall,surf}$, $T_{floor,surf}$, $T_{ceiling,surf}$, $T_{window,surf}$

Control variable – Controlled variables

Solar building example

- 2. Physical model

Most dynamic effects in building thermal behaviour are related to transient heat transfer through walls or slabs. Section 2.2 will be devoted to the problem of finding a simplified model for a multi-layer wall.

Source: Kummert 2001

- Heat transport through walls

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial^2 x} \qquad \text{with} \quad \alpha = \frac{k}{\rho c_p} \quad (\mathrm{m^2/s})$$

Heat diffusion equation: Fourier's law

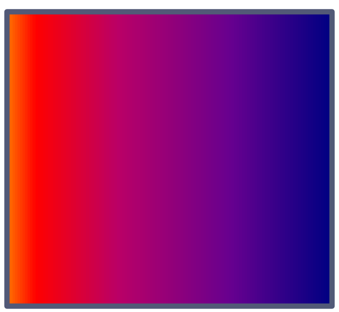
Partial Differential equation (PDE)
Continuous in space (x) and time (t)

Solar building example

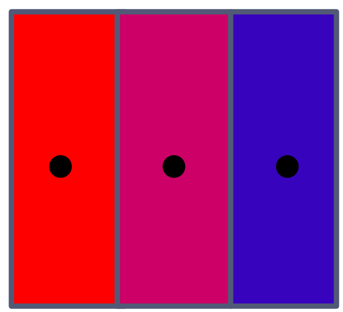
- Wall model
  - Analytical solution heat diffusion equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial^2 x} \qquad \theta(x^*) = \sum_{n=1}^{\infty} C_n \exp\left(-\varsigma_n^2 Fo\right)\cos\left(\varsigma_n x^*\right) \text{ with } Fo = \frac{\alpha t}{R^2}$$

  - Simplification 1: Discretization in space

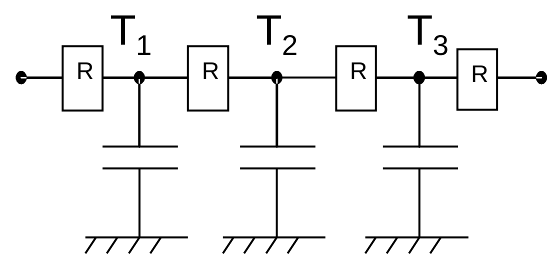


$T(x,t)$



$T_1(t)\ T_2(t)\ T_3(t)$

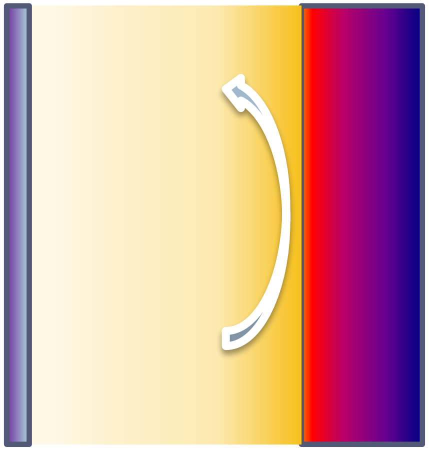
'lumped capacitances'



'RC-network representation'

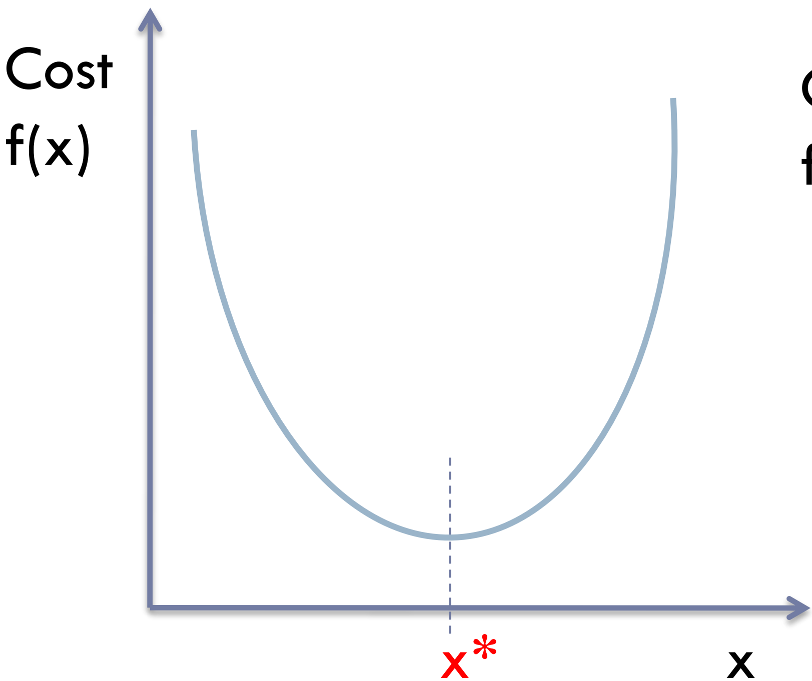
Solar building example

- Wall model
  - Boundary conditions at surface
    - Convection $\dot{Q}_{conv} = UA\left(T_{w,surf} - T_{air}\right)$ with $U = f\left(T_{air}, \dot{m}_{air}\right)$
    - Radiation $\dot{Q}_{rad,ij} = \varepsilon\sigma\left(T_{w,surf,i}{}^{4} - T_{w,surf,j}{}^{4}\right)$
    - → Nonlinear processes!



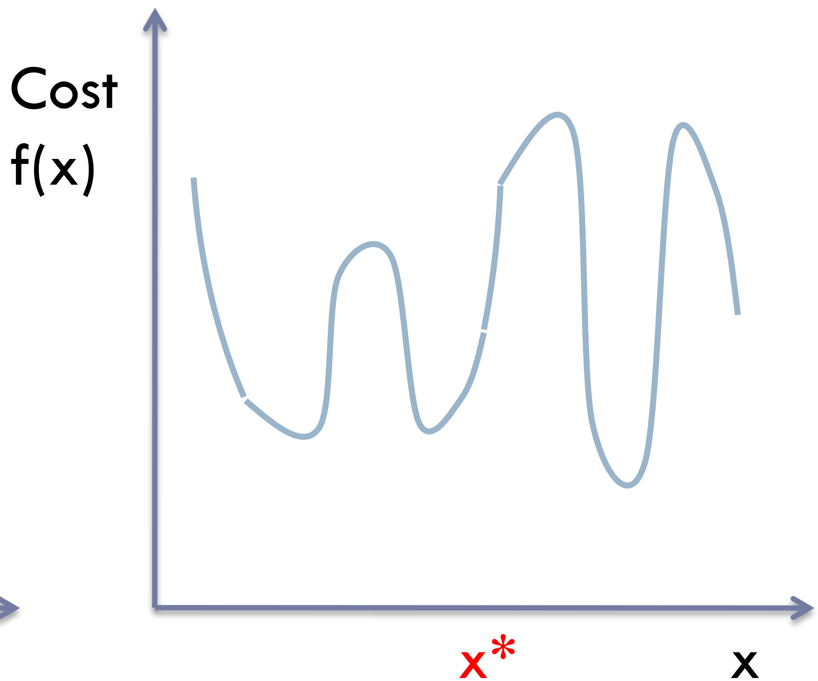
  - Simplification 2: Linearization
    - $\dot{Q}_{tot} = UA\left(T_{w,surf} - T_{air}\right)$ with $U = c^{te}$

- Convex versus non-convex optimization problems



$\min x^T A x$

$Bx <= b$



$\min f(x)$

$g(x) <= 0$

Intermezzo

- Convex versus non-convex optimization problems

$$u^* = \arg\min_{u} \int_0^T l(u(t), x(t), t)\,dt + F(x(T))$$

$$\dot{x} = f(u, x, t)$$

$$0 = g(u, x, t)$$

$$x(t) \in X$$

$$u(t) \in U$$

Nonlinear model

Convex

Convex objective function
+ Linear or second order cone constraints

✓Guaranteed convergence to global optimum
✓Proves for stability and robustness
✓Efficient algorithms

Non-convex

Non-convex objective function/ nonlinear constraints

✓Existance of local optima
✓Sensitive to choice initial values
✓No general prove for stability and robustness
✓ °Efficient algorithms

Solar building example

- 3. Proposed model structures

Our objective is to obtain a more accurate wall model than the one obtained with Laret's approach but staying at an acceptable level of complexity to be included in a simplified thermal zone model suitable for the development of a model-based controller. A model with about ten nodes or more, as the ones obtained by following the approach of (Waters and Wright, 1985), would certainly not be suitable.

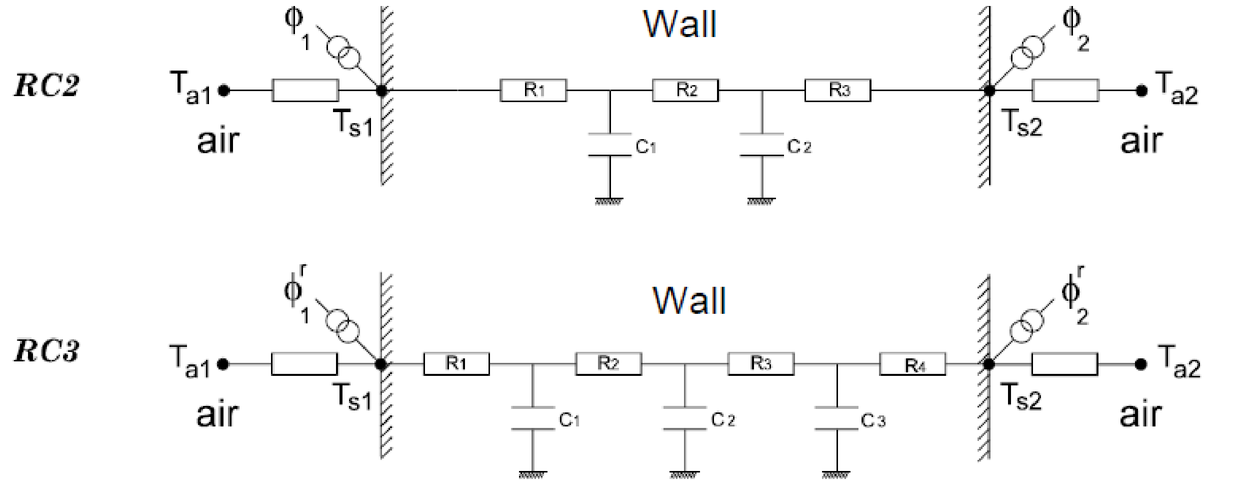
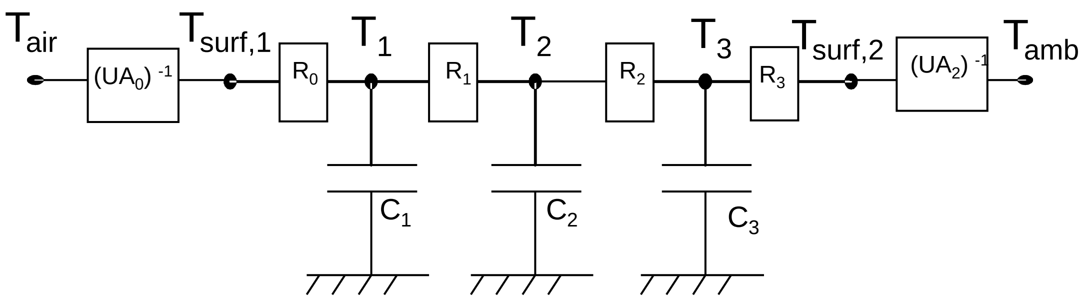


Figure 2-1: Wall lumped capacitance models with 1, 2 and 3 nodes and boundary conditions

Source: Kummert 2001

- 3. Proposed model structures
  - Model equations



$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{pmatrix} = \begin{pmatrix} -\left(\frac{1}{R_1 C_1} + \frac{1}{\left(R_0 + UA_0^{-1}\right) C_1}\right) & \frac{1}{R_1 C_1} & 0 \\ \frac{1}{R_1 C_2} & -\left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2}\right) & \frac{1}{R_2 C_2} \\ & \frac{1}{R_2 C_3} & -\left(\frac{1}{R_2 C_3} + \frac{1}{\left(R_3 + UA_3^{-1}\right) C_3}\right) \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{\left(R_0 + UA_0^{-1}\right) C_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{\left(R_3 + UA_3^{-1}\right) C_3} \end{pmatrix} \begin{pmatrix} T_{air} & T_{amb} \end{pmatrix}$$

$$\dot{T} = AT + Bu$$

- 3. Parameter estimation
  - Model parameters

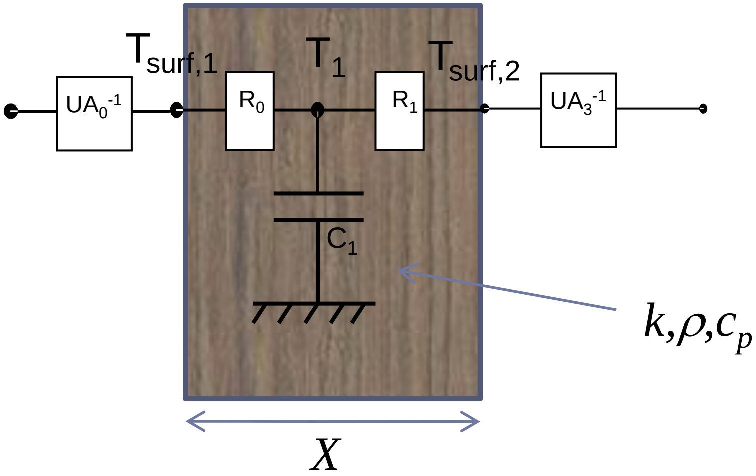
$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{pmatrix} = \begin{pmatrix} -\left(\frac{1}{R_1C_1} + \frac{1}{\left(R_0 + UA_0^{-1}\right)C_1}\right) & \frac{1}{R_1C_1} & 0 \\ \frac{1}{R_1C_2} & -\left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2}\right) & \frac{1}{R_2C_2} \\ & \frac{1}{R_2C_3} & -\left(\frac{1}{R_2C_3} + \frac{1}{\left(R_3 + UA_3^{-1}\right)C_3}\right) \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{\left(R_0 + UA_0^{-1}\right)C_1} & 0 \\ 0 & 0 \\ 0 & \frac{1}{\left(R_3 + UA_3^{-1}\right)C_3} \end{pmatrix} \begin{pmatrix} T_{air} & T_{amb} \end{pmatrix}$$

$$\begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{pmatrix} = \begin{pmatrix} -(\theta_1 + \theta_2) & \theta_1 & 0 \\ \theta_3 & -(\theta_3 + \theta_4) & \theta_4 \\ & \theta_5 & -(\theta_5 + \theta_6) \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} + \begin{pmatrix} \theta_2 & 0 \\ 0 & 0 \\ 0 & \theta_6 \end{pmatrix} \begin{pmatrix} T_{air} & T_{amb} \end{pmatrix}$$

  - Initial guess for θ
    - from known thermodynamic properties

Solar building example

- 3. Parameter estimation
  - Initial guess parameters
    - Homogeneous wall - single node
      - $C = \rho c_p X \quad \left(\frac{\mathrm{J}}{\mathrm{m}^2 K}\right)$
      - $R_0 = \frac{rX}{k} \quad \left(\frac{\mathrm{Km}^2}{W}\right)$
      - $R_1 = \frac{(1-r)X}{k} \quad \left(\frac{\mathrm{Km}^2}{W}\right)$



  - → 1 optimization variable, r

How to define?

Source: Kummert 2001

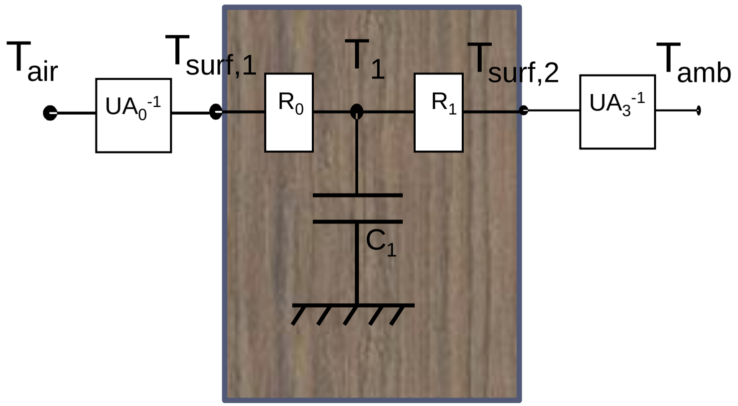
Solar building example

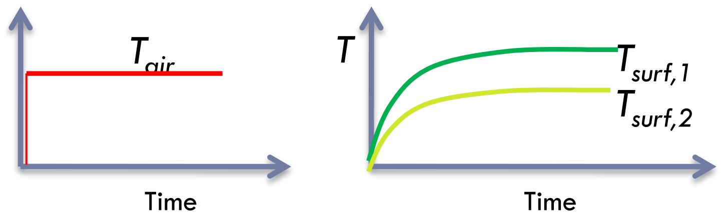
- 3. Parameter estimation
  - Optimize model parameter r
    - Needed:
      1. Real system/ Reference model
      2. Identification data set
      3. Measure for model error
      4. Optimization criterion
      5. Optimization method

Reference simulation model:
Finite Difference Model

Solar building example

- 3. Parameter estimation
  - Optimize model parameter r
    - Needed:
      1. Reference model
      2. Identification data set
      3. Measure for model error
      4. Optimization criterion
      5. Optimization method





Step input

Step response

→ Response of $T_{surf,1}$ and $T_{surf,2}$ to step change in T_{air}

Solar building example

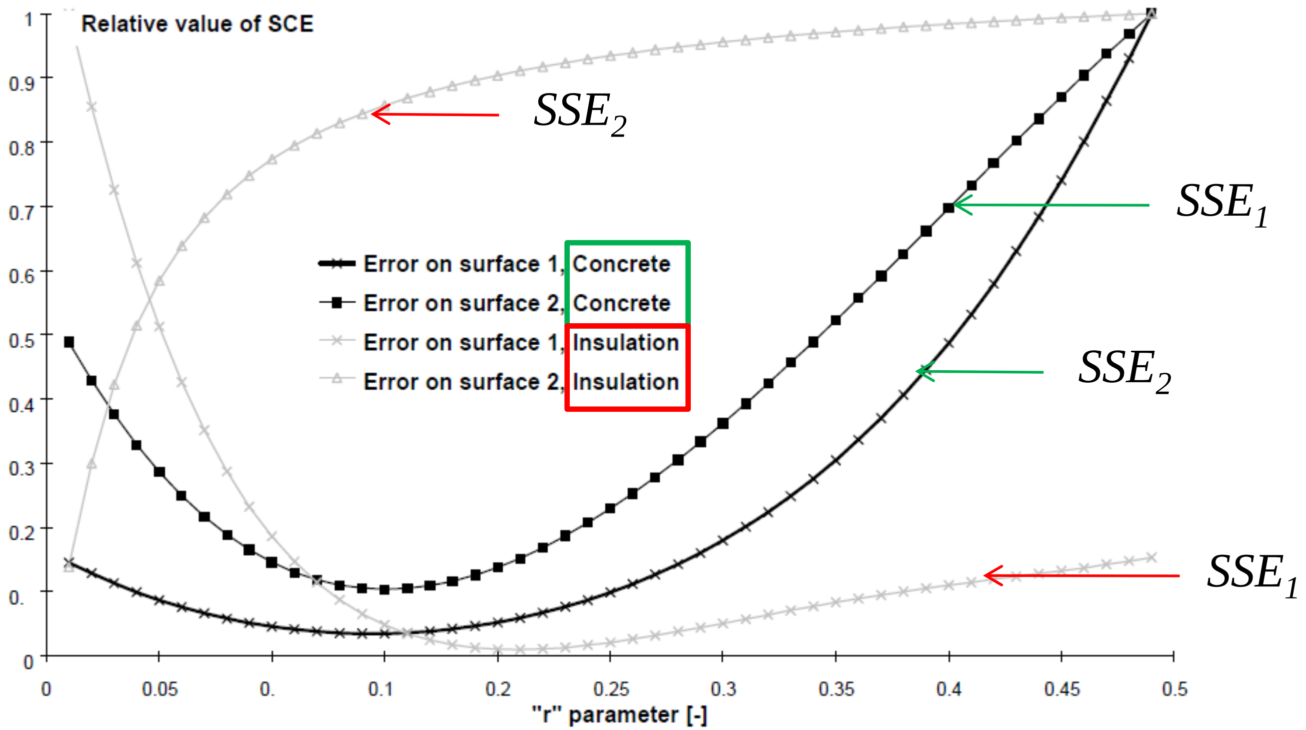
- 3. Parameter estimation
  - Optimize model parameter *r*
    - Needed:
      1. Reference model
      2. Identification data
      3. Measure for model error
      4. Optimization criterion
      5. Optimization method

$$e_1(t) = \left(T_{surf,1,\mathrm{mod}} - T_{surf,1,FD}\right)$$
$$e_2(t) = \left(T_{surf,2,\mathrm{mod}} - T_{surf,2,FD}\right)$$

$$SSE_1 = \int_0^{t_m} \left(e_1{}^T e_1\right) dt$$

$$SSE_2 = \int_0^{t_m} \left(e_2{}^T e_2\right) dt$$

- 3. Parameter estimation
  - Optimize model parameter r

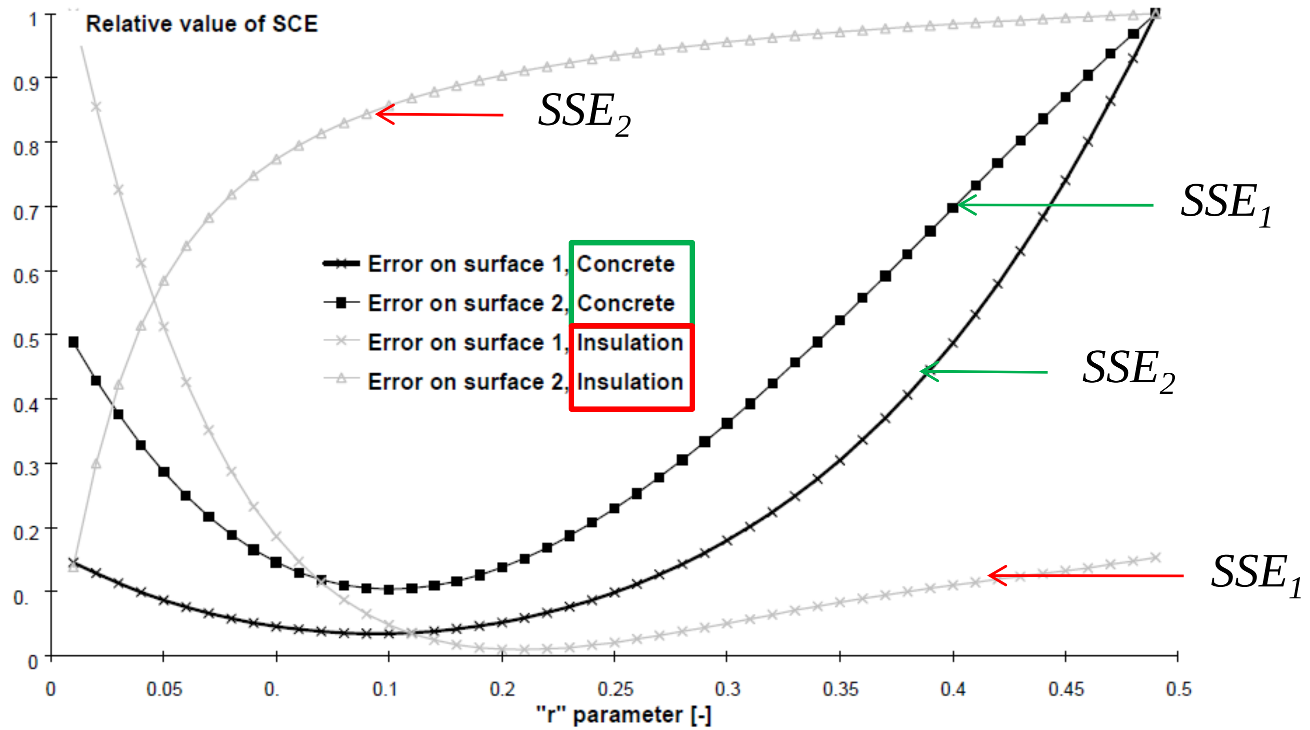


Exhaustive search

Source: Kummert 2001

Solar building example

- 3. Parameter estimation
  - Optimize model parameter r



Exhaustive search

Source: Kummert 2001

Solar building example

- 3. Parameter estimation
  - Analogously: determine parameters of
    - RC2-model
    - RC3-model

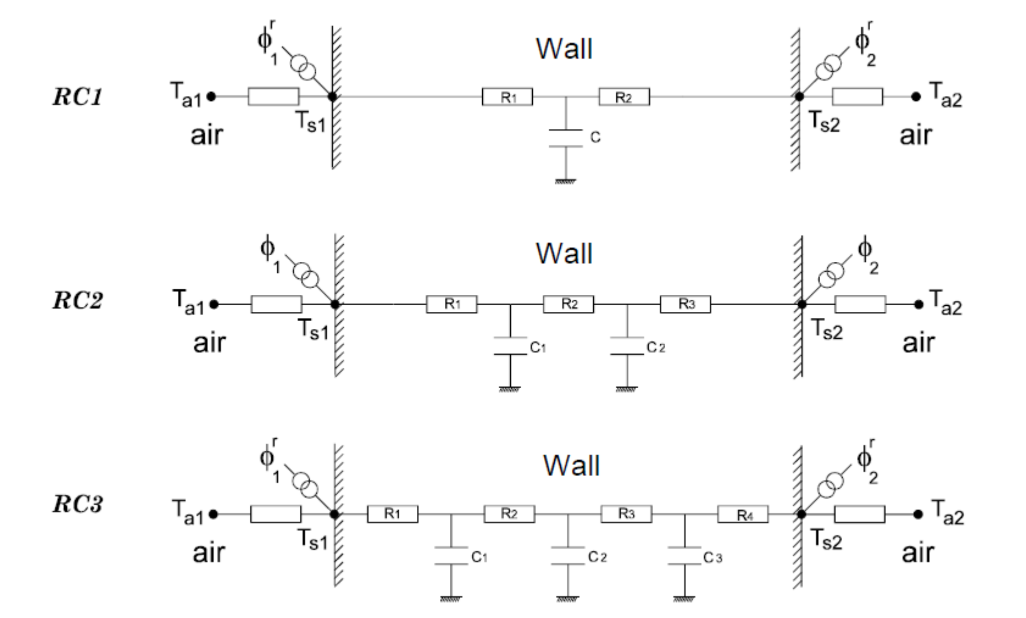
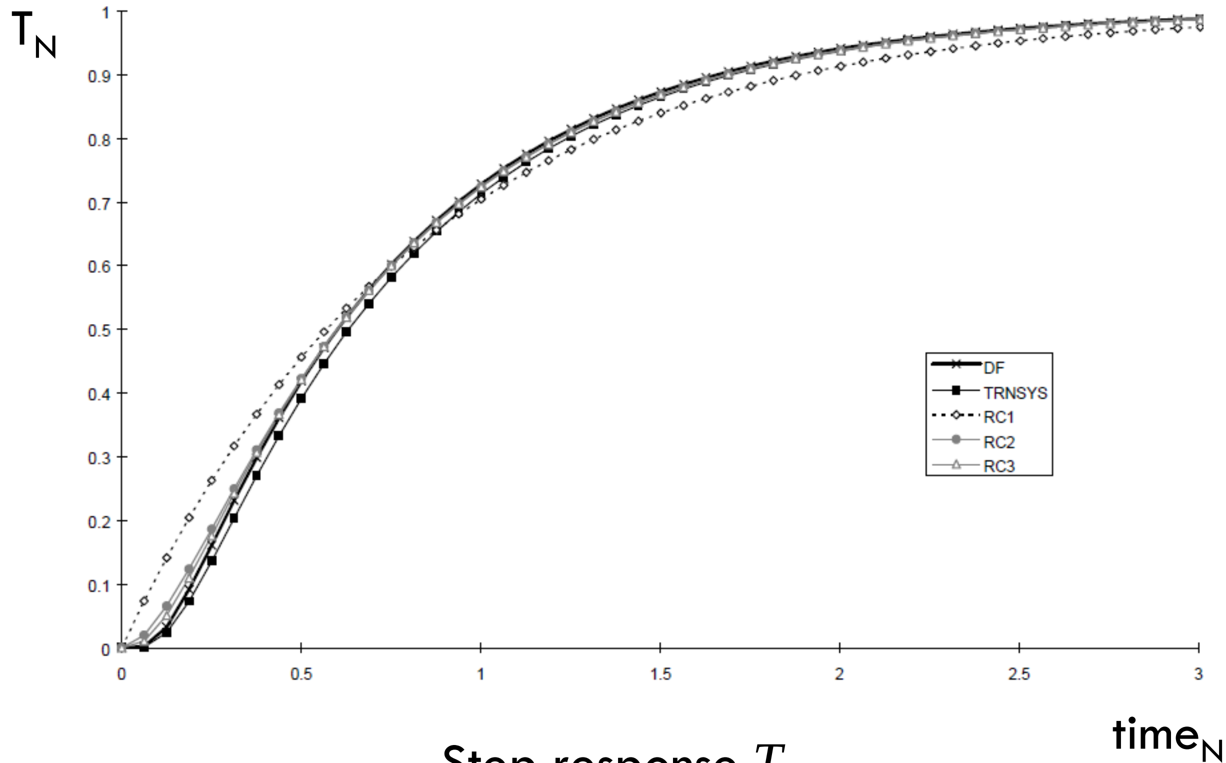


Figure 2-1: Wall lumped capacitance models with 1, 2 and 3 nodes and boundary conditions

Source: Kummert 2001

Solar building example

- 4. Model validation
  - Time domain



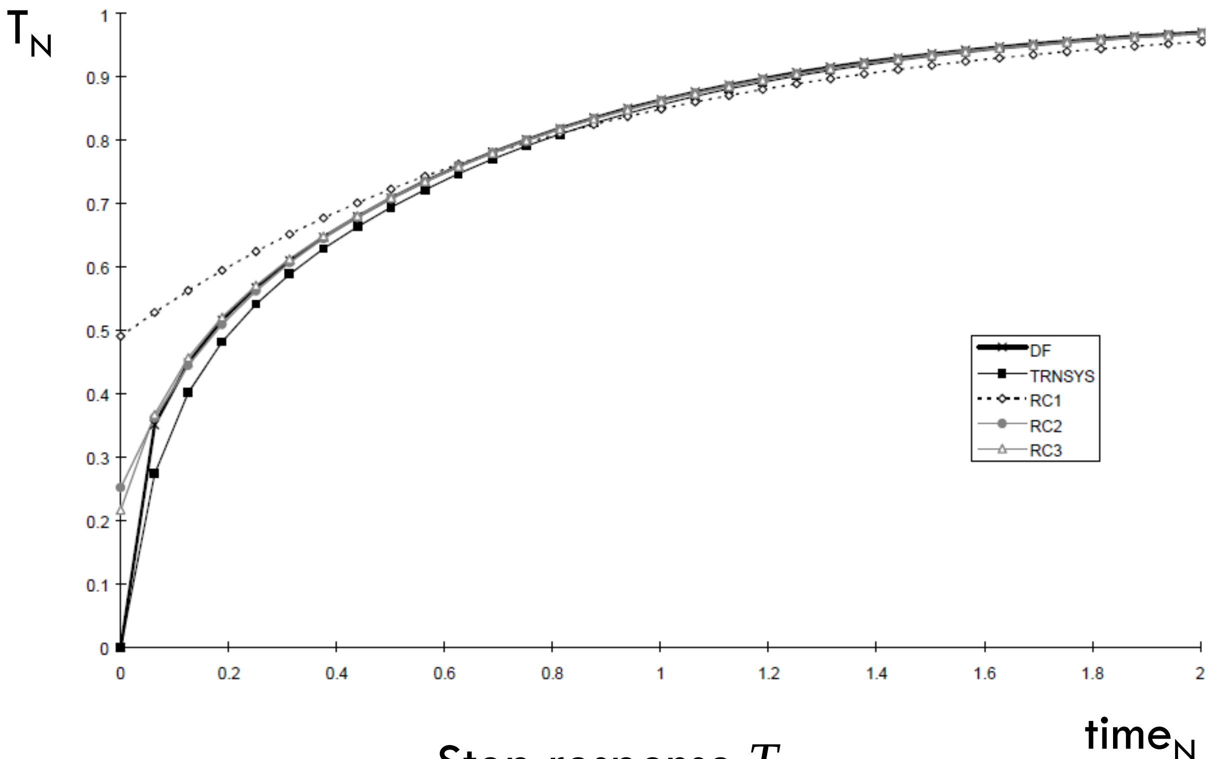
Step response $T_{surf,1}$

Concrete wall

Source: Kummert 2001

Solar building example

- 4. Model validation
  - Time domain



Step response $T_{surf,2}$

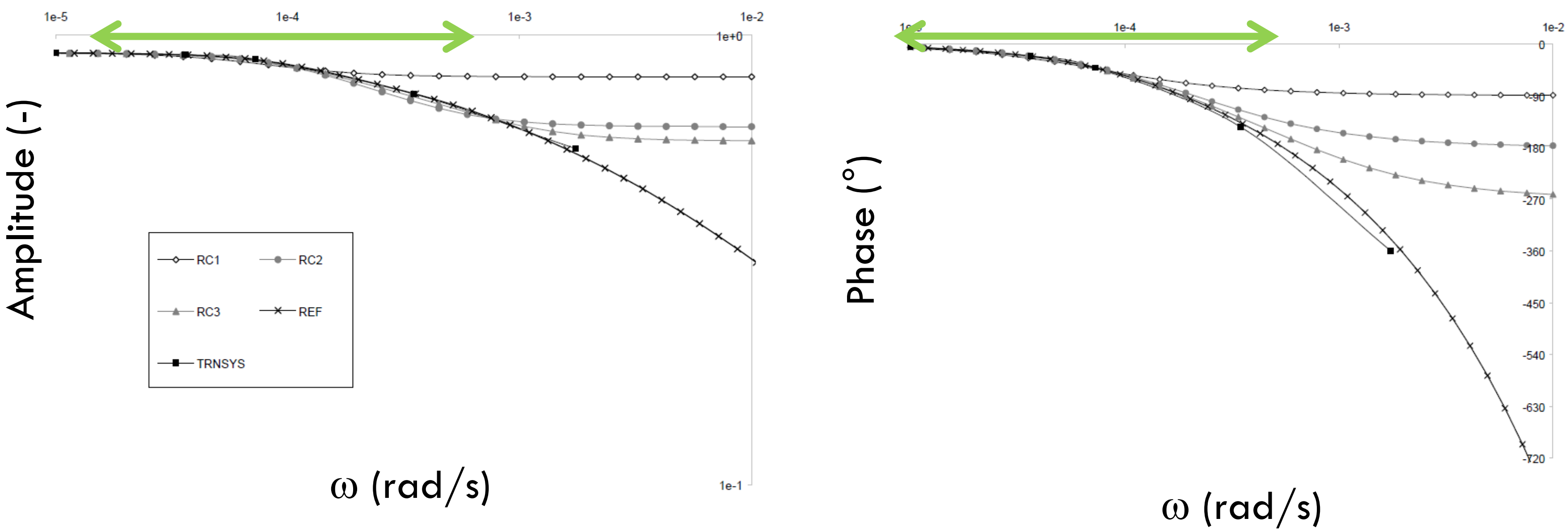
Concrete wall

Source: Kummert 2001

Solar building example

- 4. Model validation
  - Frequency domain

Frequency range of interest
- Frequency range solar gains
- Frequency range internal gains



Bode plot for $T_{surf,1}$ with respect to excitation T_{air}
Concrete wall

Source: Kummert 2001

Solar building example

- 5. Model selection
  - *"As simple as possible"*
  - *"But accurate enough"*

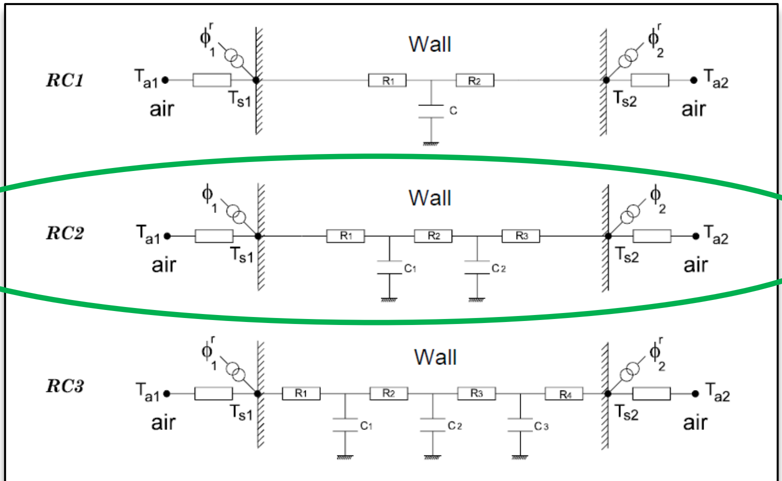
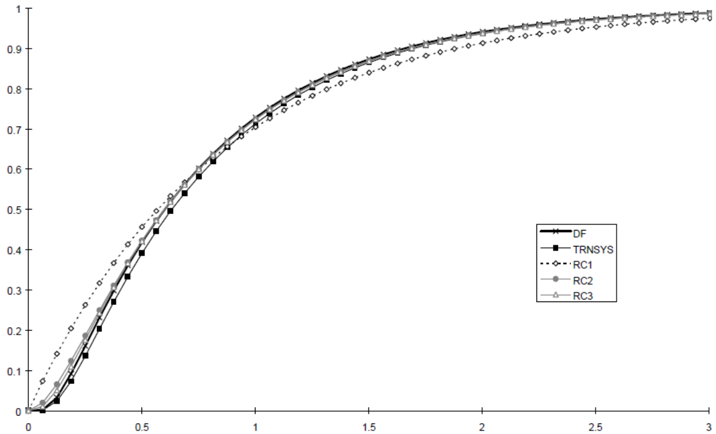
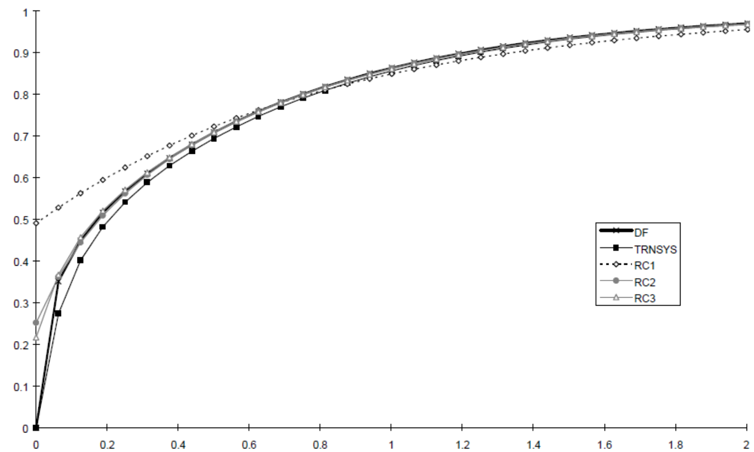


Figure 2-1: Wall lumped capacitance models with 1, 2 and 3 nodes and boundary conditions



Step response $T_{surf,1}$

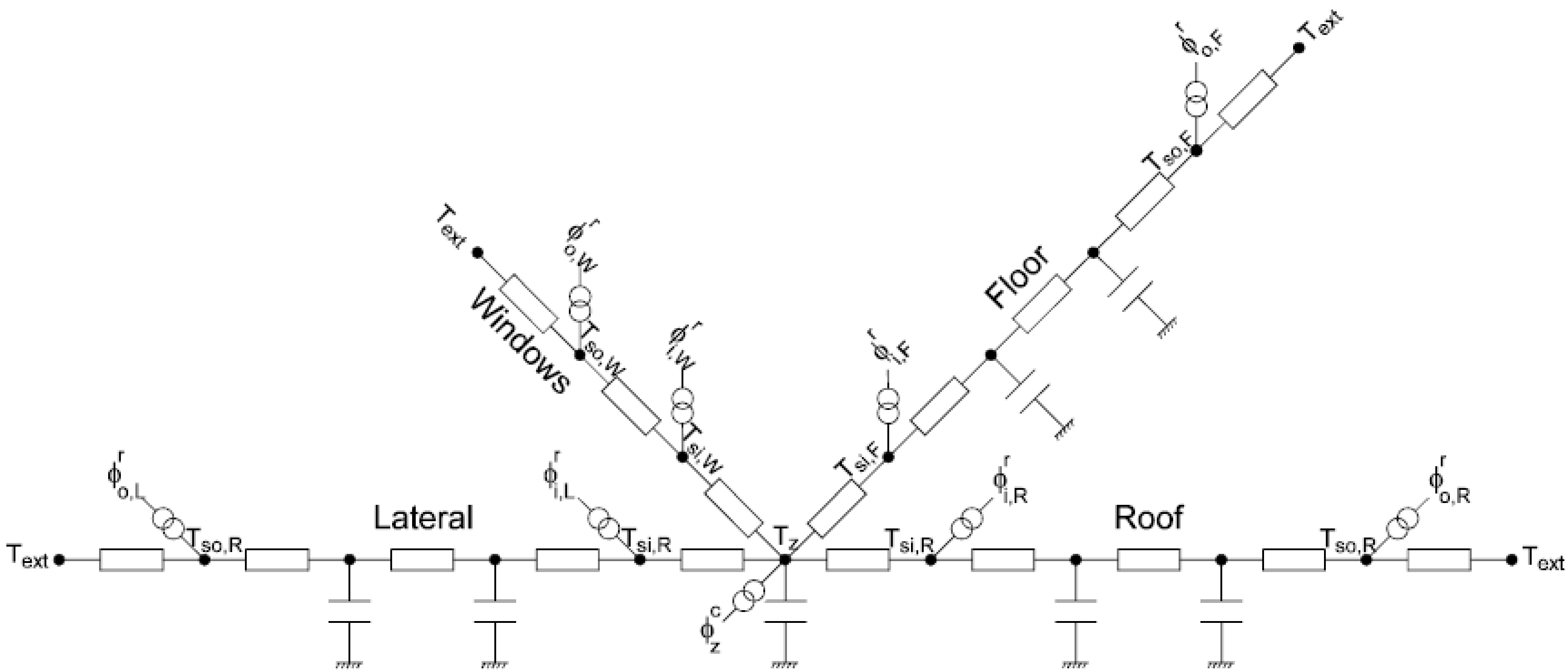


Step response $T_{surf,2}$

Source: Kummert 2001

Solar building example

- Resulting model for single-zone



Scheme of the simplified thermal zone model

Source: Kummert 2001

Solar building example

- Take home message
  - Step 1: Define control objectives
  - Step 2: Determine controlled variables and control variables

    → model inputs and outputs
  - Step 3: Write down system equations
    - Discretization in space
    - Linearization

    → model structure(s) with parameters
  - Step 4: Initial parameter estimate from physical insight
  - Step 5: Parameter estimation
  - Step 6: Model validation
  - Step 7: Model selection

References

- Kummert, M. (2001). **Contribution to the application of modern control techniques to solar buildings.** Simulation-based approach and experimental validation, Ingénieur Civil mécanicien - électricien. Liège, Fondation Université Luxembourgeoise, p. 260.

Applications in building control

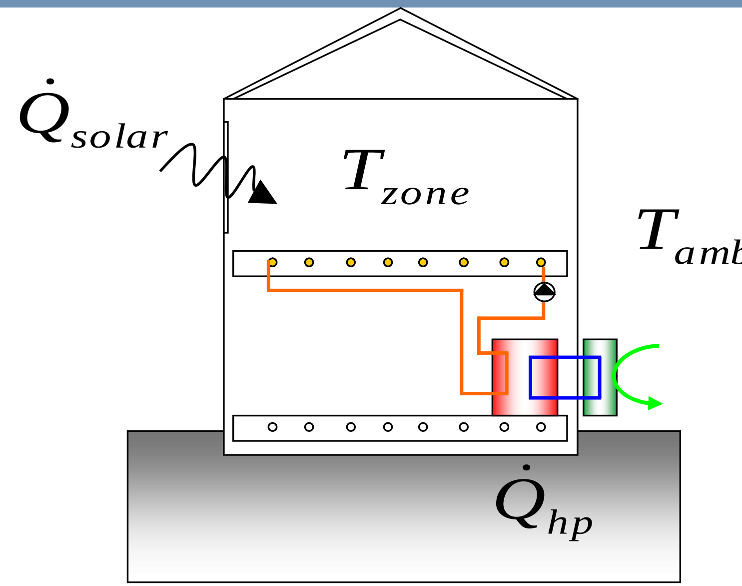
- Applications in building control
  - Heating curve control
  - MPC for heavy-weight solar building
  - MPC for heat pump system with floor heating
  - MPC for ground coupled heat pump system
  - MPC for multizone building





Online system identification example

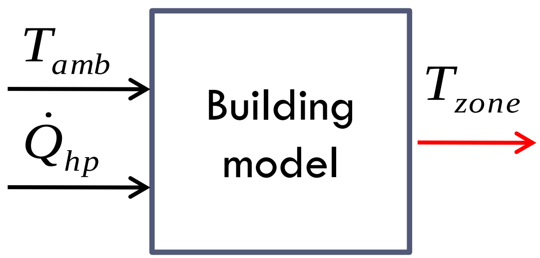
Online system identification example



- System
  - Heavy-weight building
  - Floor heating
  - Heat pump system
- Control objectives
  - Minimize thermal discomfort & electricity cost

$$J\left(\dot{Q}_{hp}\right)=\frac{\Delta t_{s,MPC}}{2}\sum_{k=0}^{N}\left\{R_k\left(\frac{c_{el,k}}{COP_k}\dot{Q}_{hp,k}\right)^2+\left(T_{zone,k}-T_{zone,k}^{ref}\right)^2\right\}$$

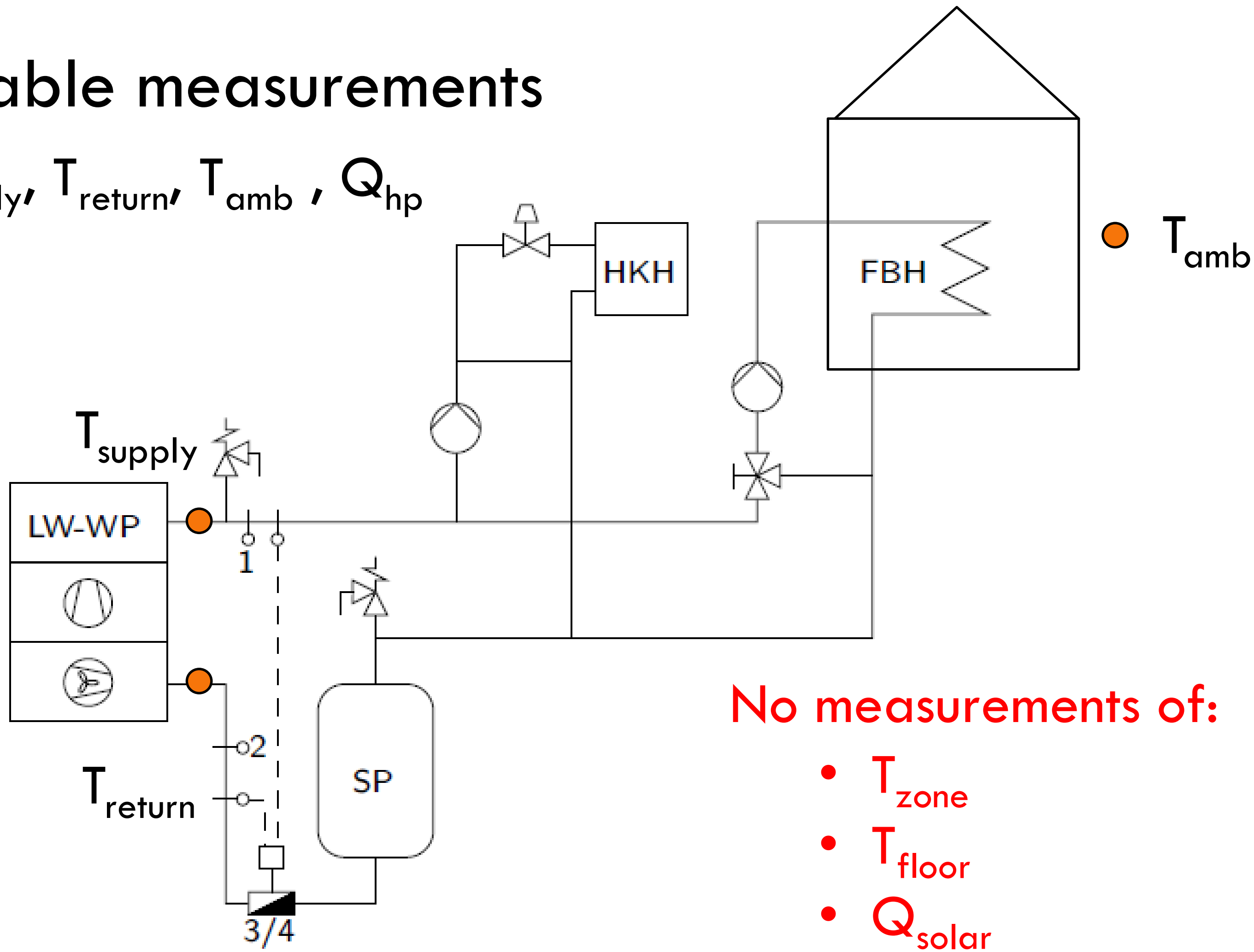
- Required model



Source: Bianchi 2006

Online system identification example

- Available measurements
  - T_{supply}, T_{return}, T_{amb} , Q_{hp}

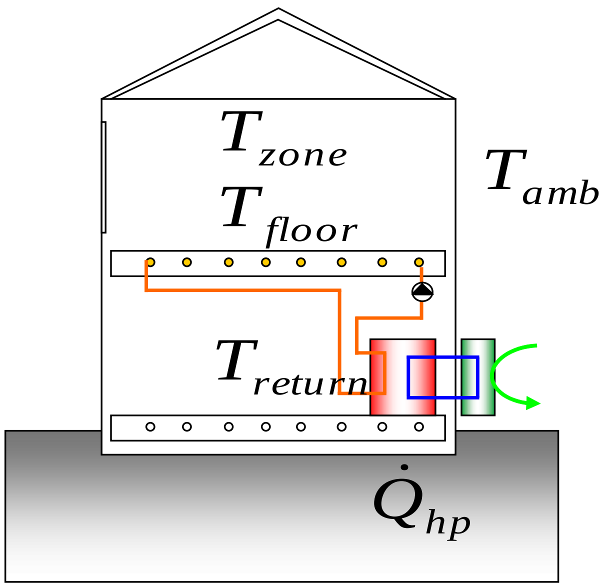


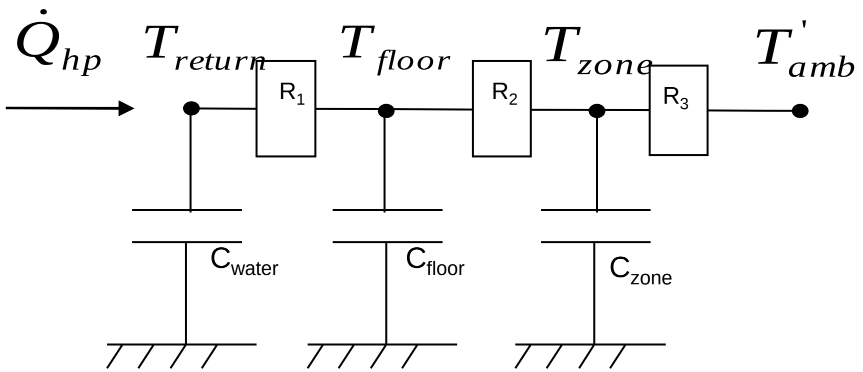
No measurements of:

- T_{zone}
- T_{floor}
- Q_{solar}

Source: Bianchi 2006

- Physical model 1



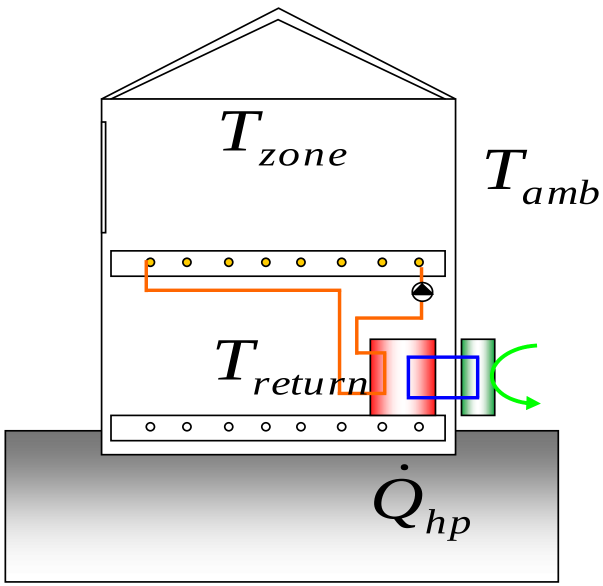


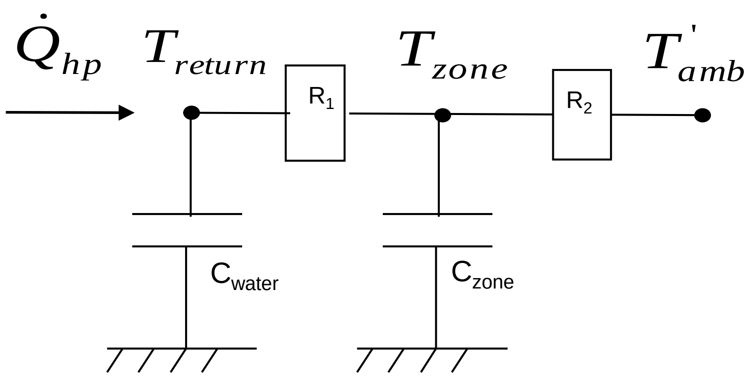
3rd order model

Source: Bianchi 2006

Online system identification example

- Physical model 2





2nd order model

Source: Bianchi 2006

Online system identification example

- Model equations
  - 3rd order model

$$\begin{bmatrix} \dot{T}_{return}(t) \\ \dot{T}_{floor}(t) \\ \dot{T}_{zone}(t) \end{bmatrix} = \begin{bmatrix} -\frac{k_{wf}}{\rho_w c_w V_w} & \frac{k_{wf}}{\rho_w c_w V_w} & 0 \\ \frac{k_{wf}}{m_f c_f} & -\frac{k_{wf}+k_{fz}}{m_f c_f} & \frac{k_{fz}}{m_f c_f} \\ 0 & \frac{k_{fz}}{\tau_b k_b} & -\frac{k_{fz}+k_b}{\tau_b k_b} \end{bmatrix} \begin{bmatrix} T_{return}(t) \\ T_{floor}(t) \\ T_{zone}(t) \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{\rho_w c_w V_w} \\ 0 & 0 \\ \frac{1}{\tau_b} & 0 \end{bmatrix} \begin{bmatrix} T_{amb}(t) + \Delta T_{stat} \\ \dot{Q}_{hp}(t) \end{bmatrix}$$

6 parameters

  - 2nd order model

$$\begin{bmatrix} \dot{T}_{return}(t) \\ \dot{T}_{zone}(t) \end{bmatrix} = \begin{bmatrix} -\frac{k_{wz}}{\rho_w c_w V_w} & \frac{k_{wz}}{\rho_w c_w V_w} \\ \frac{k_{wz}}{k_b \tau_b} & -\frac{k_{wz}+k_b}{k_b \tau_b} \end{bmatrix} \begin{bmatrix} T_{return}(t) \\ T_{zone}(t) \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{\rho_w c_w V_w} \\ \frac{1}{\tau_b} & 0 \end{bmatrix} \begin{bmatrix} T_{amb}(t) + \Delta T_{stat} \\ \dot{Q}_{hp}(t) \end{bmatrix}$$

4 parameters

Source: Bianchi 2006

Online system identification example

- Parameter estimation
  - Problems!
    - Real data

      → measurement errors on:
      - input variables
      - output variables
    - Non-measured inputs:
      - Solar gains
      - Internal gains
    - Non-measured states

      → more unknown variables

noise

Unmodelled disturbances

Proces simplification

Source: Bianchi 2006

Online system identification example

- Requirements for identification
  - Based on online measurement data
  - Robust with respect to:
    - Modeling errors
    - Noise
  - Recognition of solar gains
  - Physically meaningful parameters
  - Good prediction of building thermal behaviour for a time horizon of 24 hours

Source: Bianchi 2006

Implications of existance of noise

- In ideal case
  - No noise
  - Perfect model knowledge
- Then
  - If excitation is persistent
    - Parameters are uniquely defined
    - Solution not dependent on excitation signal
  - Number of equations needed = number of parameters

Deterministic problem

Source: Bianchi 2006

Implications of existance of noise

- Deterministic solution
  - Example 2nd order model (MISO)

$$\begin{bmatrix} \dot{T}_{return}(t) \\ \dot{T}_{zone}(t) \end{bmatrix} = \begin{bmatrix} -\frac{k_{wz}}{\rho_w c_w V_w} & \frac{k_{wz}}{\rho_w c_w V_w} \\ \frac{k_{wz}}{k_b \tau_b} & -\frac{k_{wz}+k_b}{k_b \tau_b} \end{bmatrix} \begin{bmatrix} T_{return}(t) \\ T_{zone}(t) \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{\rho_w c_w V_w} \\ \frac{1}{\tau_b} & 0 \end{bmatrix} \begin{bmatrix} T_{amb}(t) + \Delta T_{stat} \\ \dot{Q}_{hp}(t) \end{bmatrix}$$

$$T_R(t) - 2T_R(t-1) + T_R(t-2) = \begin{bmatrix} (T_R(t-1) - T_R(t-2))t_s \\ (Q_{hp}(t-1) - Q_{hp}(t-2))t_s \\ T_R(t-2) - T_{amb}(t-2)t_s^2 \\ Q_{hp}(t-2)t_s^2 \end{bmatrix}^T \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

with

$$\theta_1 = -\frac{k_{wz}}{\rho_w c_w V_w} - \frac{k_{wz}+k_b}{k_b \tau_b} \qquad \theta_2 = \frac{1}{\rho_w c_w V_w} \qquad \theta_3 = \frac{k_{wz}}{\rho_w c_w V_w \tau_b} \qquad \theta_4 = \frac{k_{wz}+k_b}{\rho_w c_w V_w \tau_b k_b}$$

Source: Bianchi 2006

Implications of existance of noise

- Deterministic solution
  - Example 2nd order model (MISO)

$$\begin{aligned} \varphi^T(3).\theta &= y(3) \\ \vdots \quad &= \quad \vdots \\ \varphi^T(6).\theta &= y(6) \end{aligned} \;\Rightarrow\; \varphi^T.\theta = y \;\Rightarrow\; \theta^* = (\varphi'\varphi)^{-1}\varphi' y$$

Source: Bianchi 2006

Implications of existance of noise

- In real case

Proces simplification	Unmodelled disturbances	noise

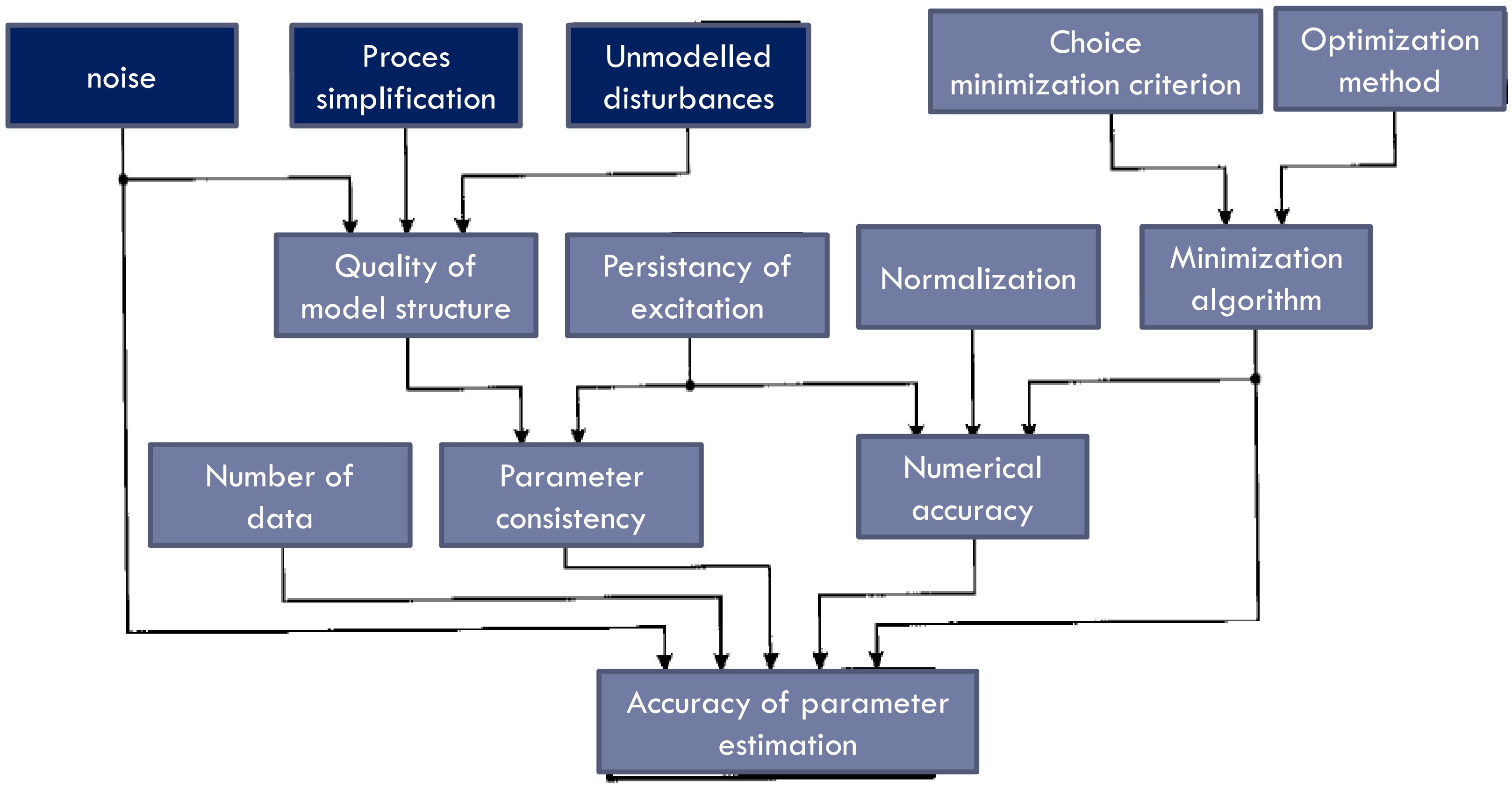
- Then
  - Solution of parameter estimation NOT uniquely defined
  - Solution depends on
    - Model structure
    - Excitation signal for identification
    - Measurement quality
    - Parameter estimation procedure

Stochastic problem

Source: Bianchi 2006

Implication of existance of noise

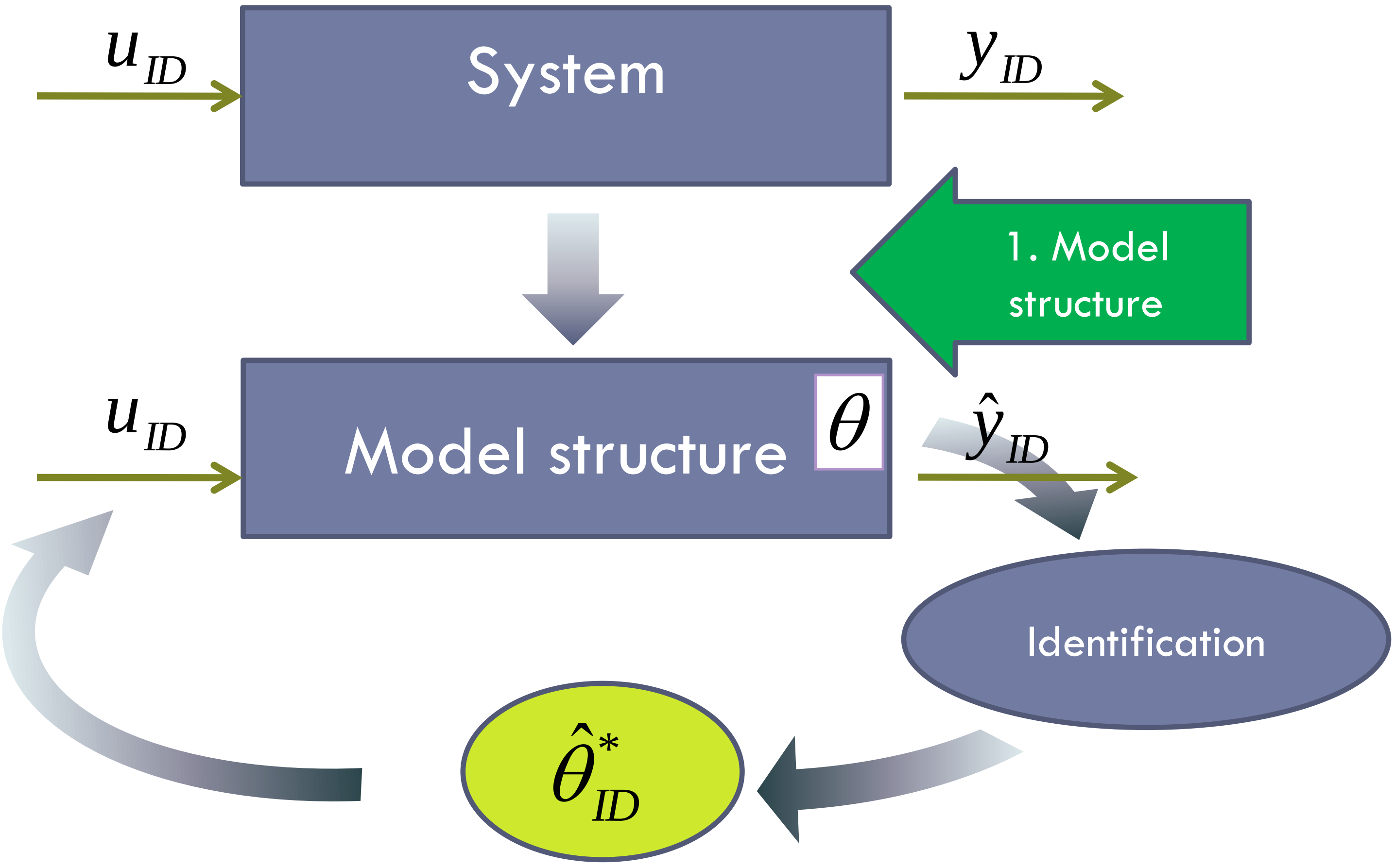
- Factors determining parameter estimation accuracy



Source: Bianchi 2006

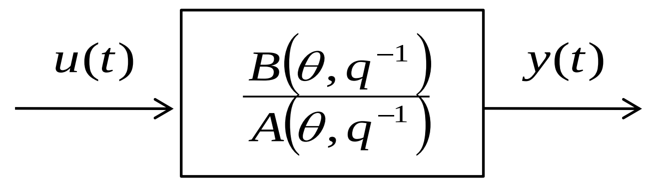
Implication of existance of noise

- 1. Model structure

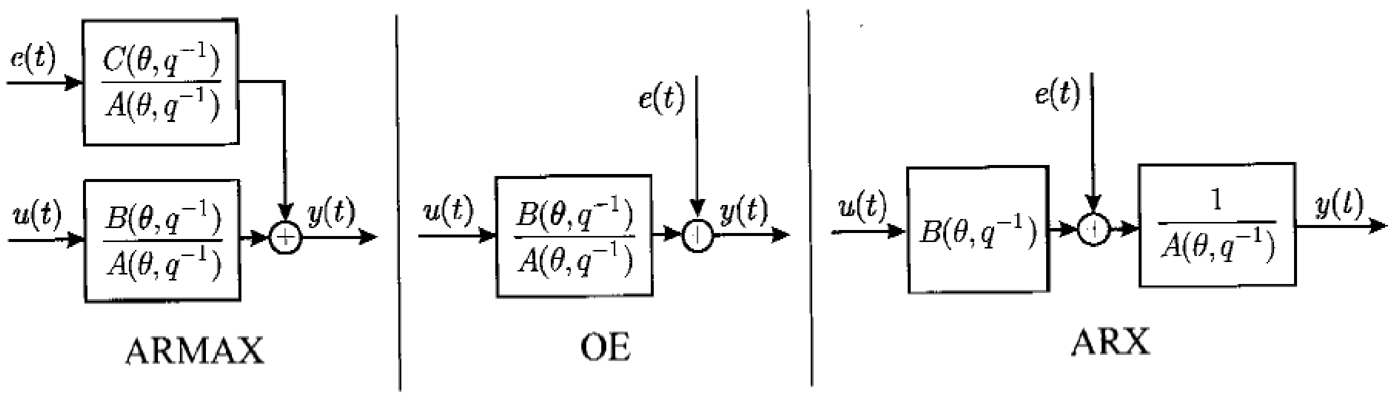


Implications of existance of noise

- 1. Model structure
  - Deterministic part
    - Example input-output representation



  - Deterministic + stochastic part
    - Figure out where noise comes into system

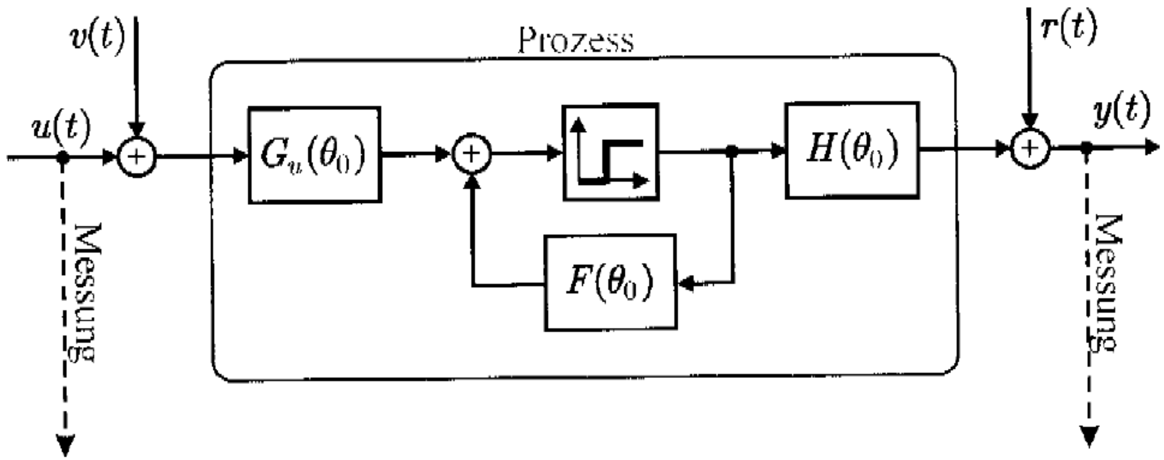


Signal flow for different input-output models (SISO)

Source: Bianchi 2006

Implications of existance of noise

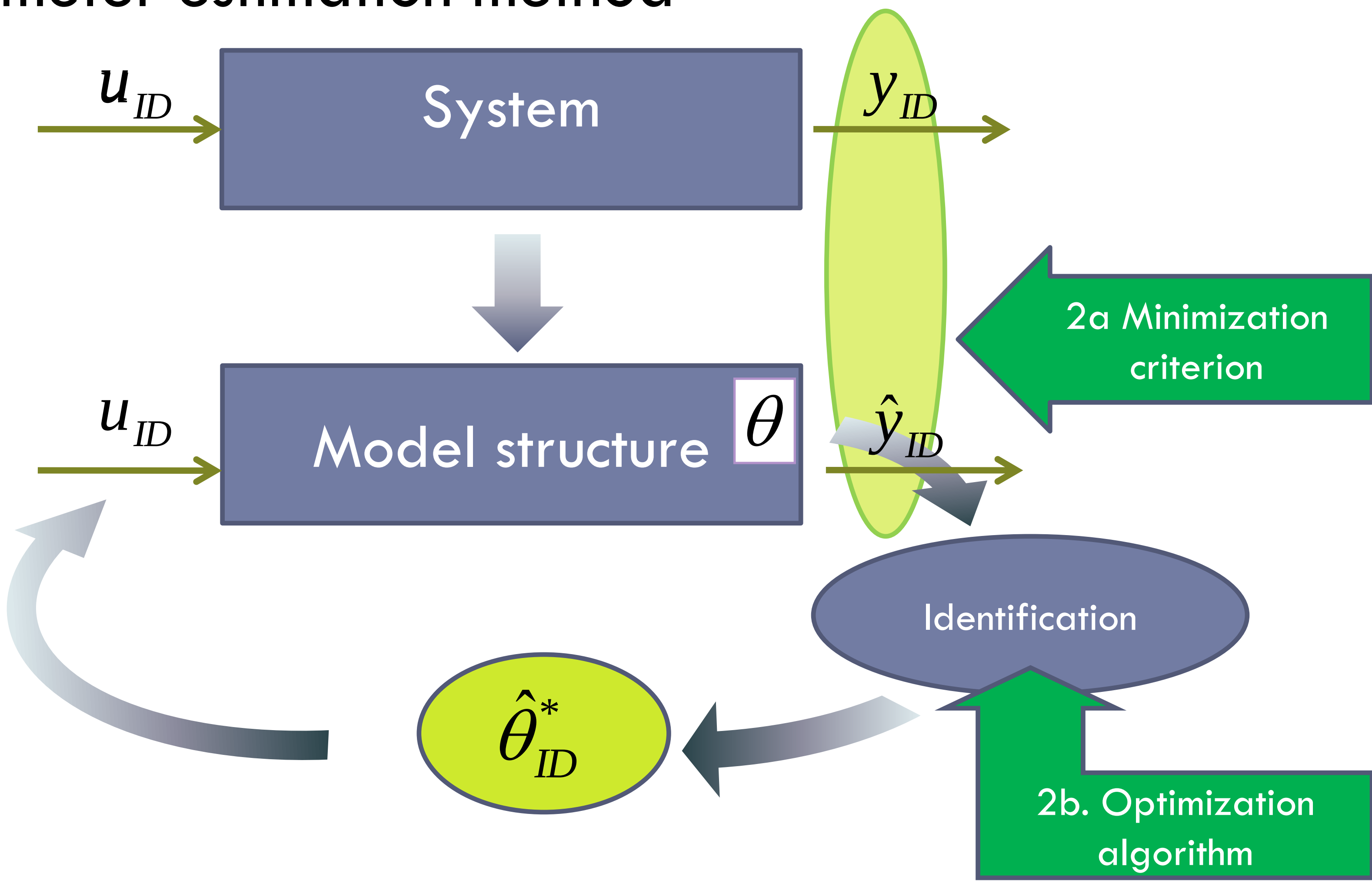
- 1. Model structure
  - Deterministic + stochastic part
    - Example state-space representation



Signal flow in state-space representation

Implication of existance of noise

- Parameter estimation method



Implications of existance of noise

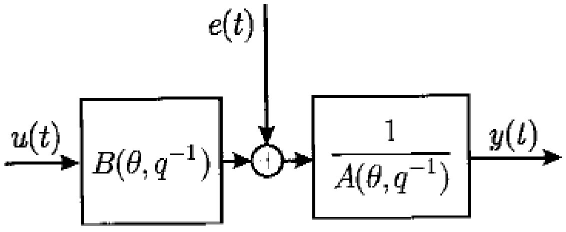
- 2. Parameter estimation method
  - Discussed methods
    - Linear Regression (LR)
    - Prediction Error Method (PEM)
    - Bayesian Approach

Source: Bianchi 2006

Implications of existance of noise

- Linear Regression (LR)
  - Model structure

$$y(t) = \phi^T(t)\theta + e(t)$$



ARX

  - Minimization of Squared Sum of Errors (SSE)

$$Q_N = \sum_1^N e(t)e^T(t) = \sum_1^N \left(y(t) - \phi^T(t)\hat{\theta}\right)\left(y(t) - \phi^T(t)\hat{\theta}\right)^T$$

  - Analytical solution

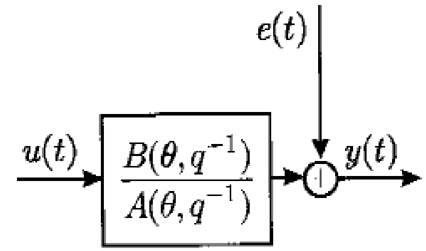
$$\hat{\theta} = \left[\sum_1^N \phi(t)\phi^T(t)\right]^{-1} \sum_1^N \phi(t)y(t)$$

Source: Bianchi 2006

Implications of existance of noise

Prediction Error Method (PEM)

Output Error model structure (OE)



OE

- Covariance of prediction error

$$Q_N = \sum_{1}^{N} \varepsilon(t,\theta)\varepsilon^T(t,\theta)$$

- Prediction error

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t|t-1) = H_{IO}^{-1}(q^{-1},\theta)\left[y(t) - G_{IO}(q^{-1},\theta)\cdot u(t)\right] \approx e(t)$$

- Optimal predictor

$$\hat{y}(t|t-1,\theta) = L_1(q^{-1},\theta)\cdot y(t) + L_2(q^{-1},\theta)\cdot u(t)$$

with

$$L_1(q^{-1},\theta) = I - H_{IO}^{-1}(q^{-1},\theta)$$

$$L_1(q^{-1},\theta) = H_{IO}^{-1}(q^{-1},\theta)\cdot G_{IO}(q^{-1},\theta).$$

Source: Bianchi 2006

Implications of existance of noise

- Cost function
  - $V_N(\theta) = \Lambda_N(\theta)^{-1} trace(Q_N(\theta))$
  - $V_N(\theta) = \det(Q_N(\theta))$
  - $\mathrm{Lik}(\theta) = \dfrac{1}{(2\pi)^{N.n_y/2}\left[\det(\Lambda(\theta))\right]^{N/2}} \exp\left(-\dfrac{1}{2}\sum_{t=1}^{N} \varepsilon^T(t,\theta)\Lambda_N^{-1}(\theta)\varepsilon(t,\theta)\right)$

$p(y(1)...y(N)|\theta)$

- Optimal parameter estimation $\hat{\theta}^*$

$$\hat{\theta}^* = \arg\min_{\theta} V_N(\theta)$$

  - $V_N(\theta)$ is <u>nonlinear</u> in the parameters θ
  - → Iterative search
  - → Importance initial values

Source: Bianchi 2006

Implications of existance of noise

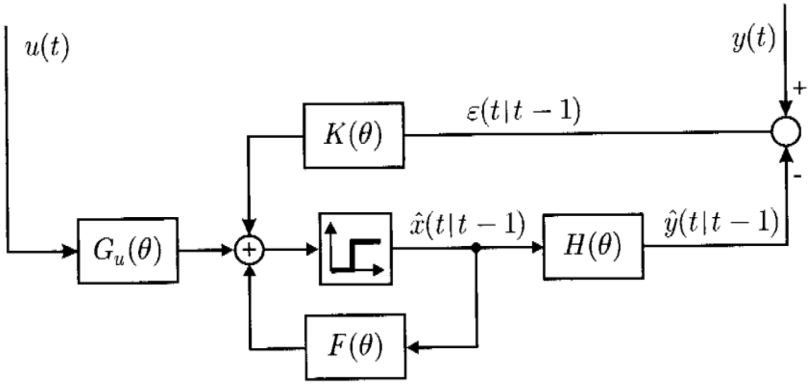
- Prediction Error Method

  State-space model structure

  - Covariance of Estimation error

$$\Sigma(t+1|t) = E\left\{[x(t+1) - \hat{x}(t+1|t)][x(t+1) - \hat{x}(t+1|t)]^T\right\}.$$

  - Optimal predictor



$$\left.\begin{aligned}
\hat{x}(t+1|t) &= F(\theta)\hat{x}(t|t-1) + G_u(\theta)u(t) + K(t)[y(t) - H(\theta)\hat{x}(t|t-1)] \\
\Sigma(t+1|t) &= F(\theta)\Sigma(t|t-1)F^T(\theta) + G_v(\theta)R_v(\theta)G_v^T(\theta) - K(t)Q(t)K^T(t) \\
K(t) &= \left[F(\theta)\Sigma(t|t-1)H^T(\theta) + G_v(\theta)R_{vr}(\theta)\right]Q^{-1}(t) \\
Q(t) &= H(\theta)\Sigma(t|t-1)H^T(\theta) + R_r(\theta),
\end{aligned}\right\}$$

Kalman filter

Source: Bianchi 2006

Implications of existance of noise

- Bayesian approach
  - A posteriori probability function

$$p\left(\theta \middle| y(1)\ldots y(N)\right) = \frac{p\left(y(1)\ldots y(N) \middle| \theta\right) p(\theta)}{p\left(y(1)\ldots y(N)\right)}$$

  - Option 1: Maximize the A Posteriori Probability (MAP)
  - Option 2: Use Kalman-filter, assuming θ to be state variable

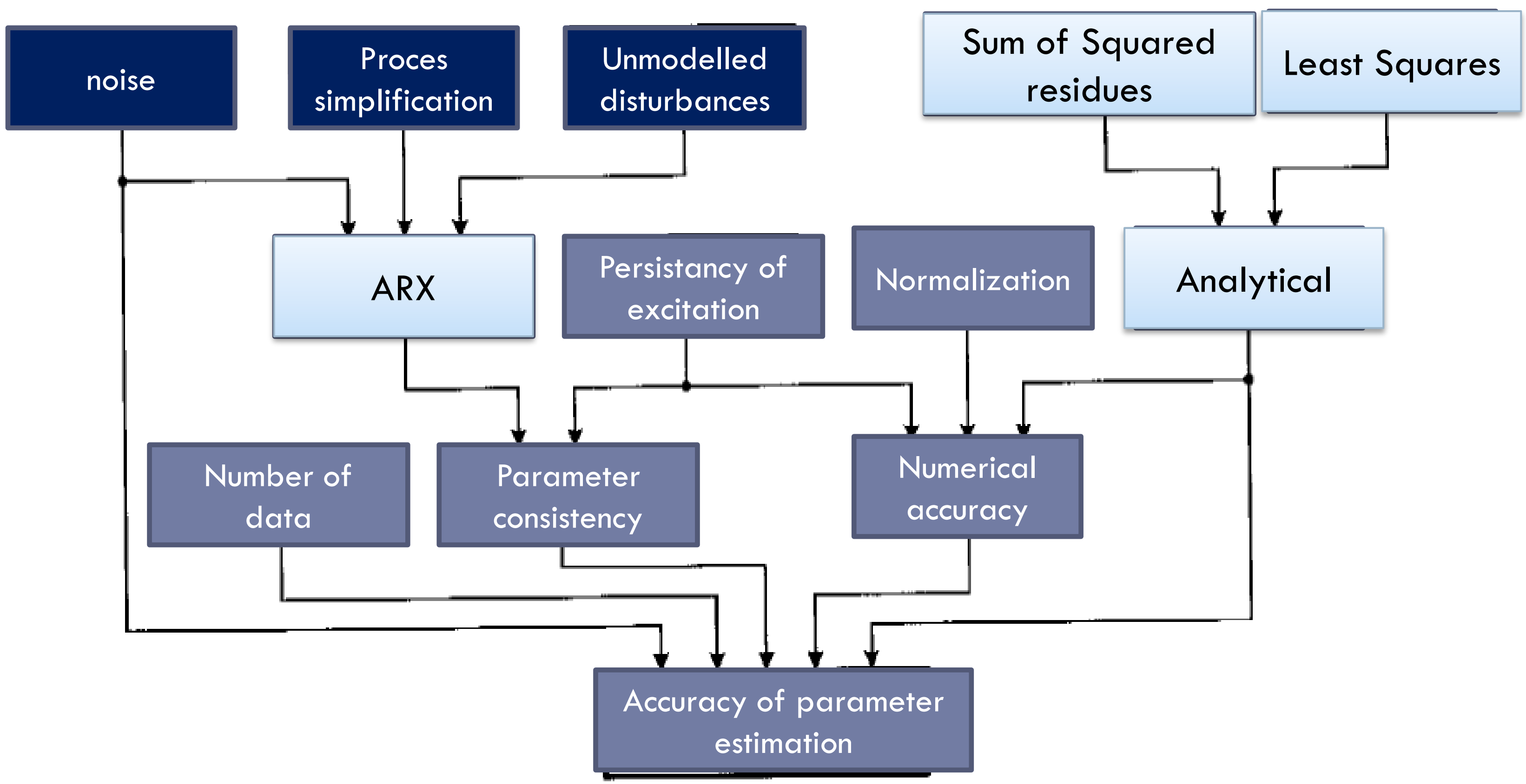
$$\left.\begin{aligned} \begin{bmatrix} x(t+1) \\ \theta \end{bmatrix} &= \begin{bmatrix} F(\theta) \cdot x(t) + G_u(\theta) \cdot u(t) \\ \theta \end{bmatrix} \\ y(t) &= H(\theta) \cdot x(t). \end{aligned}\right\} \text{Augmented state}$$

  - → Nonlinear → Extended Kalman-filter

Source: Bianchi 2006

Implication of existance of noise

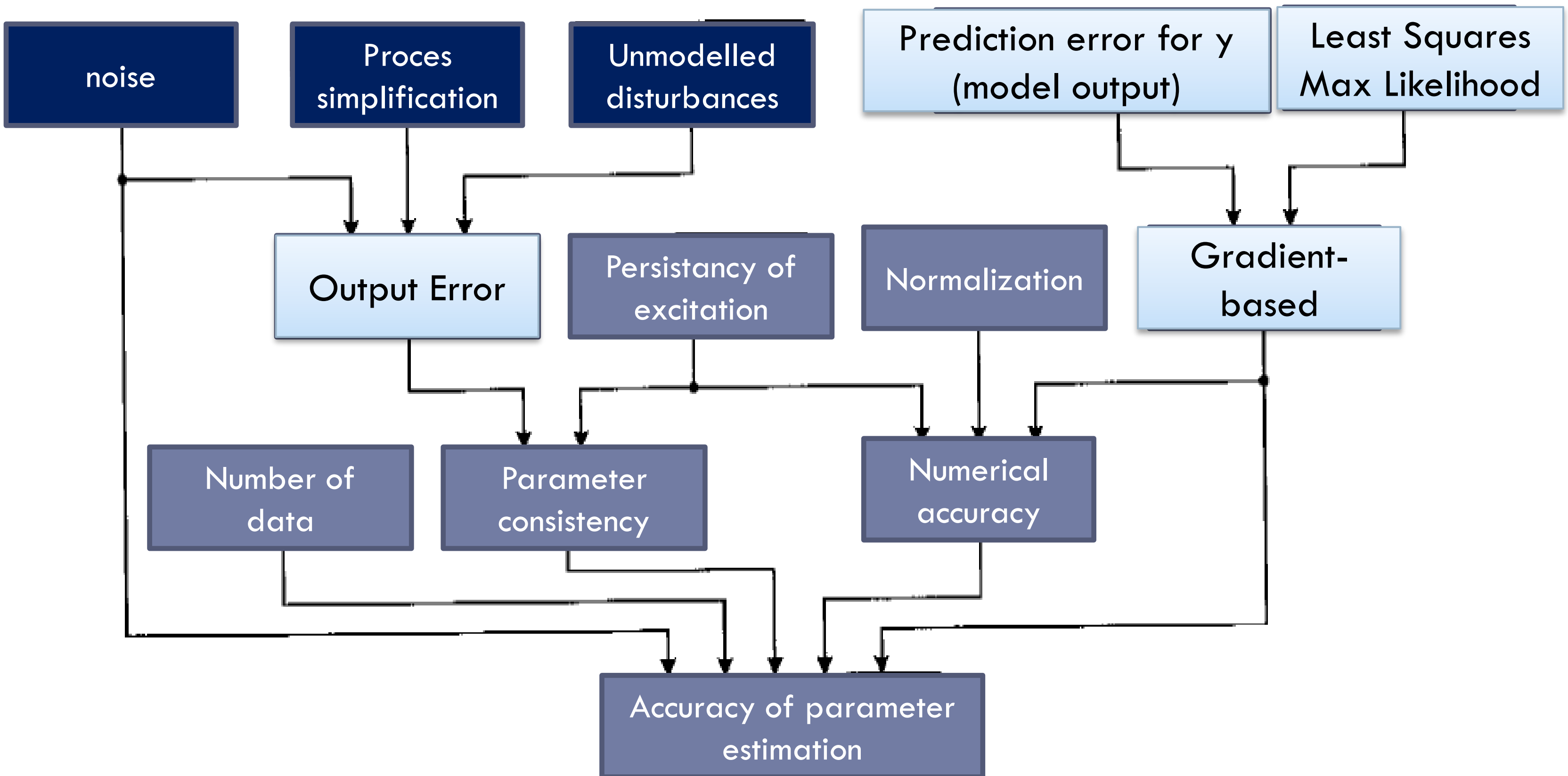
- Factors determining parameter estimation accuracy



Source: Bianchi 2006

Implication of existance of noise

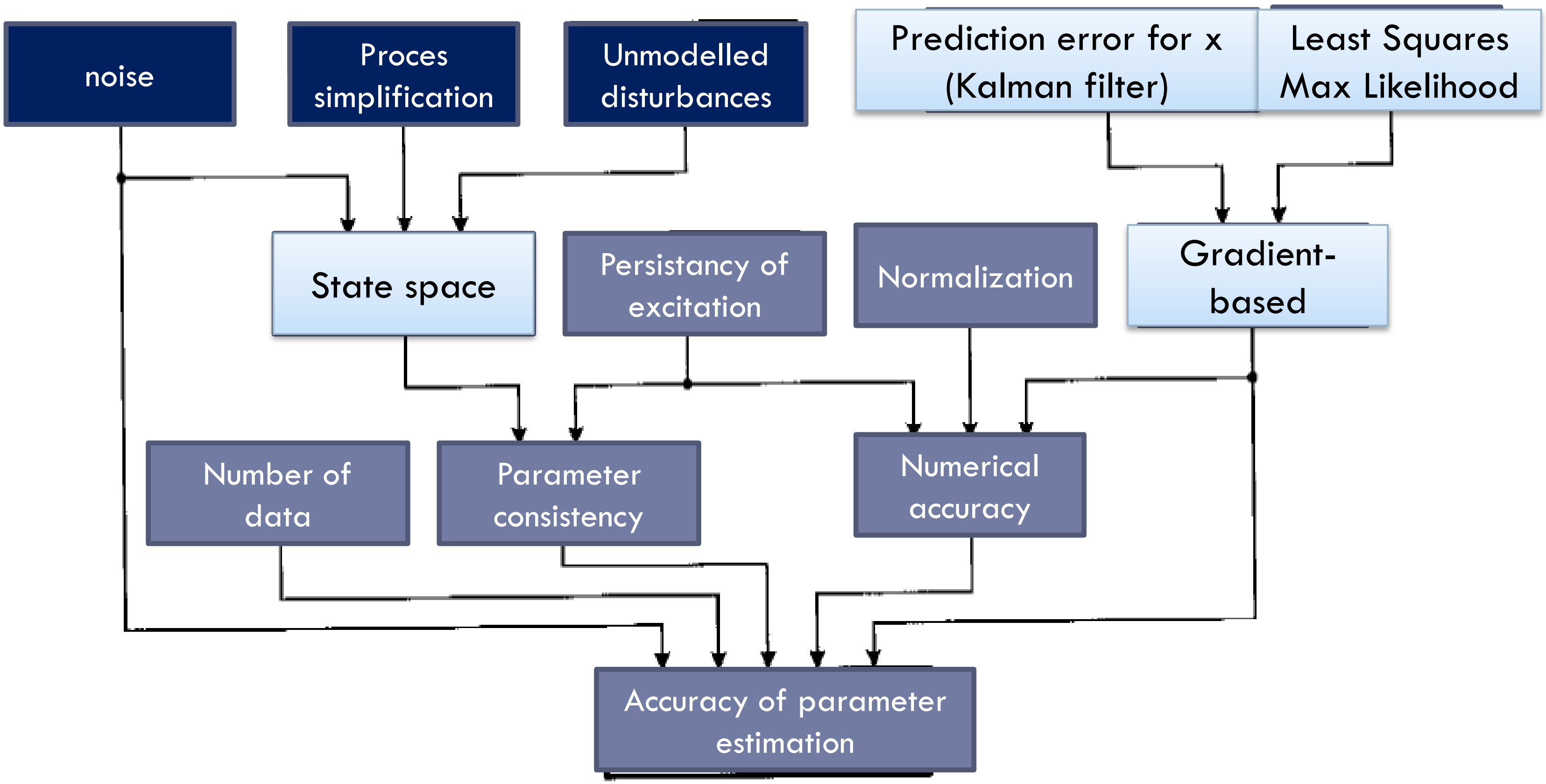
- Factors determining parameter estimation accuracy



Source: Bianchi 2006

Implication of existance of noise

- Factors determining parameter estimation accuracy



Source: Bianchi 2006

Online system identification example

- Question
  - Which combination of…
    - Model order
    - Model structure
    - Parameter estimation procedure
  - … yields
    - Physically meaningfull parameters
    - Robust against
      - Measurement noise
      - Process simplification
      - Unmeasured solar gains

Source: Bianchi 2006

Online system identification example

- Methodology
  - 'Measurement data'
    - 200 Monte-Carlo simulations with 3rd order model
      - Different T_{amb}-profiles
      - Same Q_{solar}-profiles
      - Given variance for measurement noise
        - Temperatures: variance 0.01 °C², read-out accuracy: 0.1°C
        - Heating power: variance 0.25 kW², read-out accuracy:0.2 kW
      - Simulation time: 21 days
      - Sampling time: 5 minutes
    - → For each set of data: parameter estimation

Source: Bianchi 2006

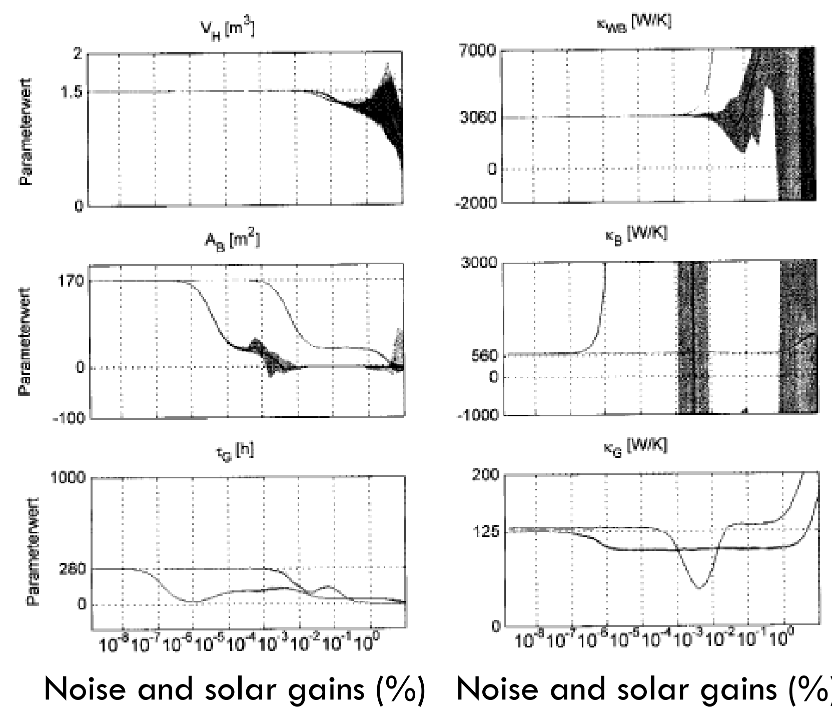
Online system identification example

- Methodology
  - Parameter estimation methods
    - Offline
      - Least Squares
      - Prediction Error Method – Output Error
      - Prediction Error Method – Kalman Filter Predictor
    - Online
      - Recursive least squares
      - Recursive maximum likelihood
      - Extended Kalman filter
      - Extended Kalman filter + RML as predictor

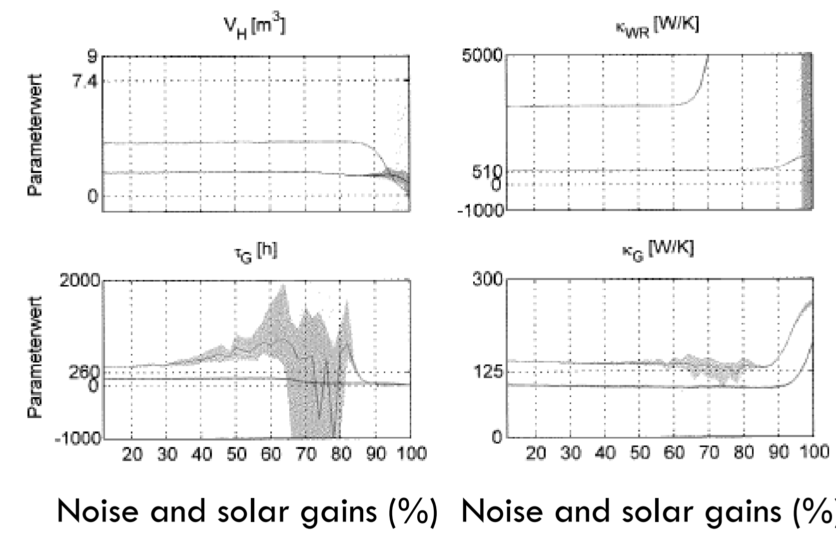
Source: Bianchi 2006

Online system identification example

- Offline: results least squares



3rd order model
Least Squares

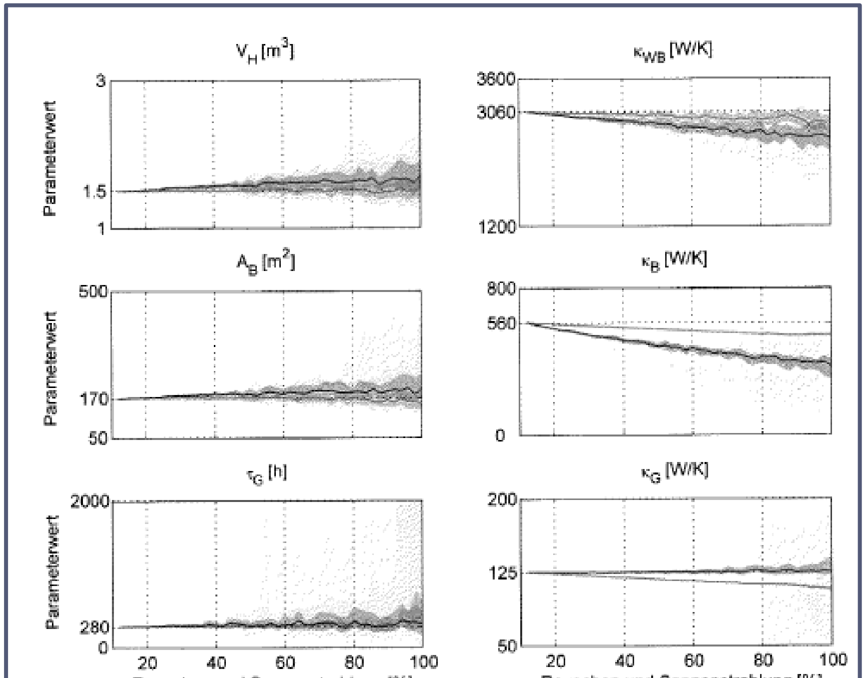


2nd order model
Least Squares

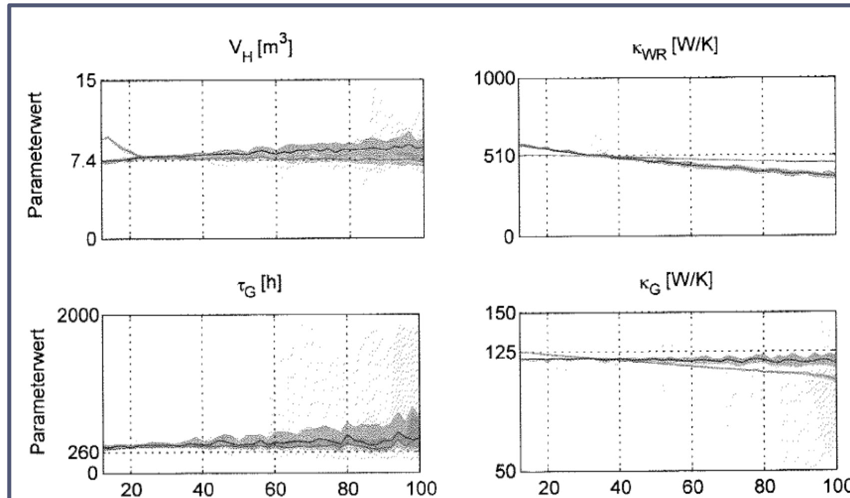
Source: Bianchi 2006

Online system identification example

- Offline: results PEM – Output Error



Noise and solar gains (%) Noise and solar gains (%)



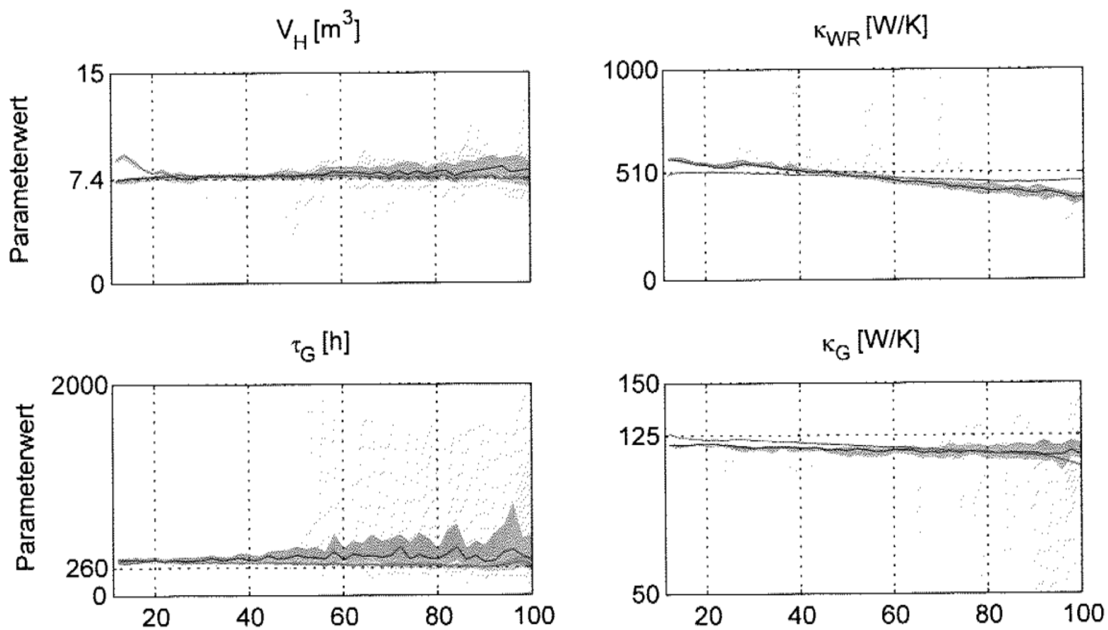
Noise and solar gains (%) Noise and solar gains (%)

3rd order model	2nd order model
Prediction Error Method	Prediction Error Method
Output error model structure	Output error model structure
Maximum Likelihood	Maximum Likelihood

Source: Bianchi 2006

Online system identification example

- Offline: results PEM – Kalman filter predictor

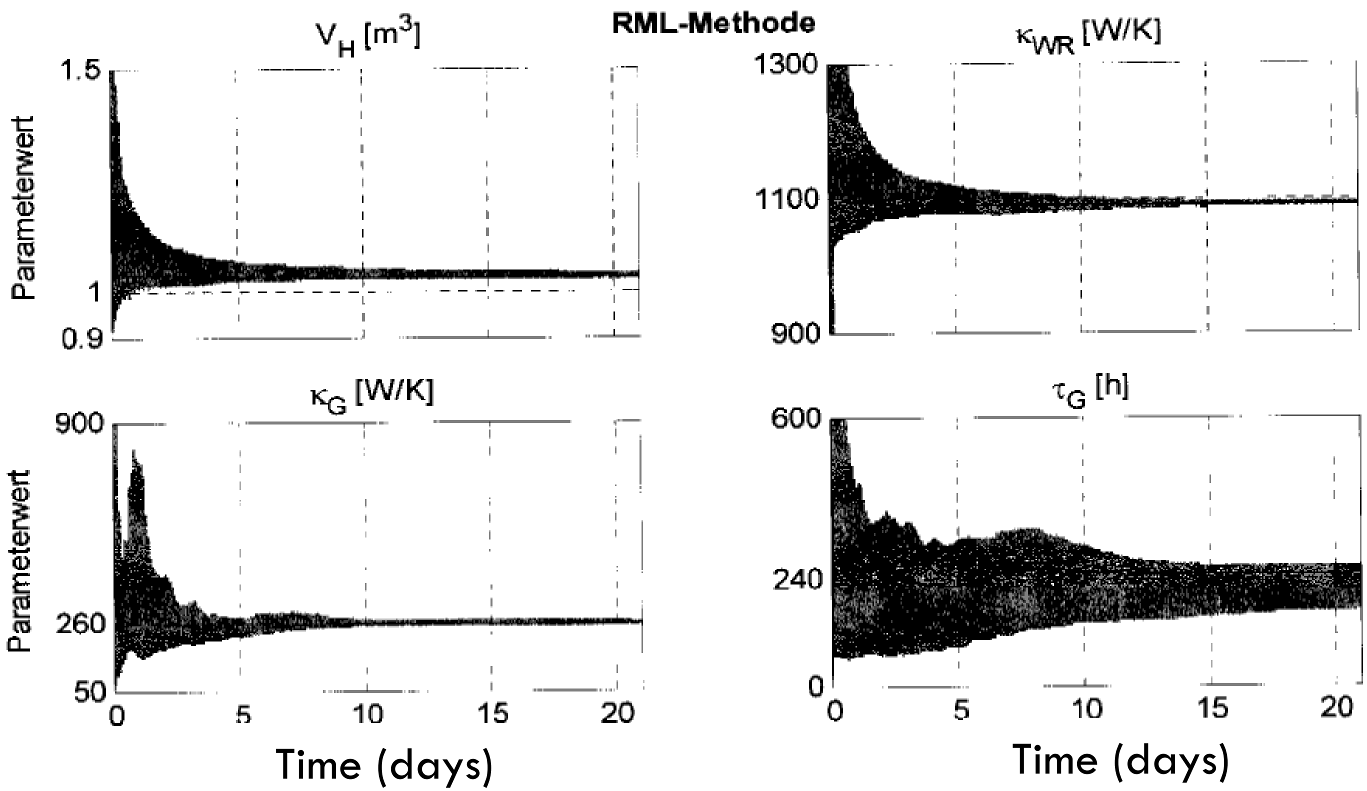


2nd order model
Prediction Error Method
Kalman filter predictor
Maximum Likelihood

Source: Bianchi 2006

Online system identification example

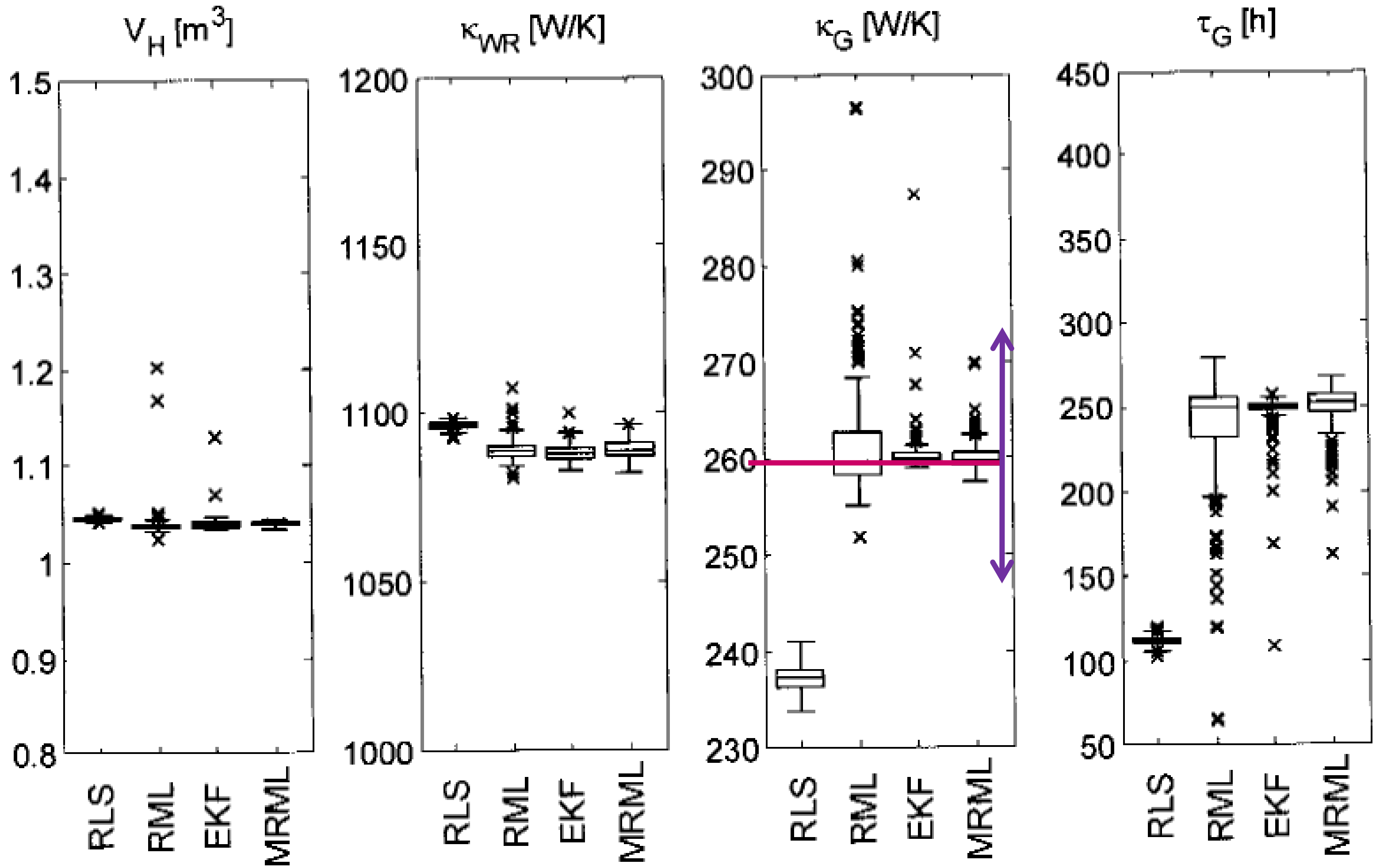
- **Online: results**
  - Example: Recursive Maximum Likelihood (RML)



Source: Bianchi 2006

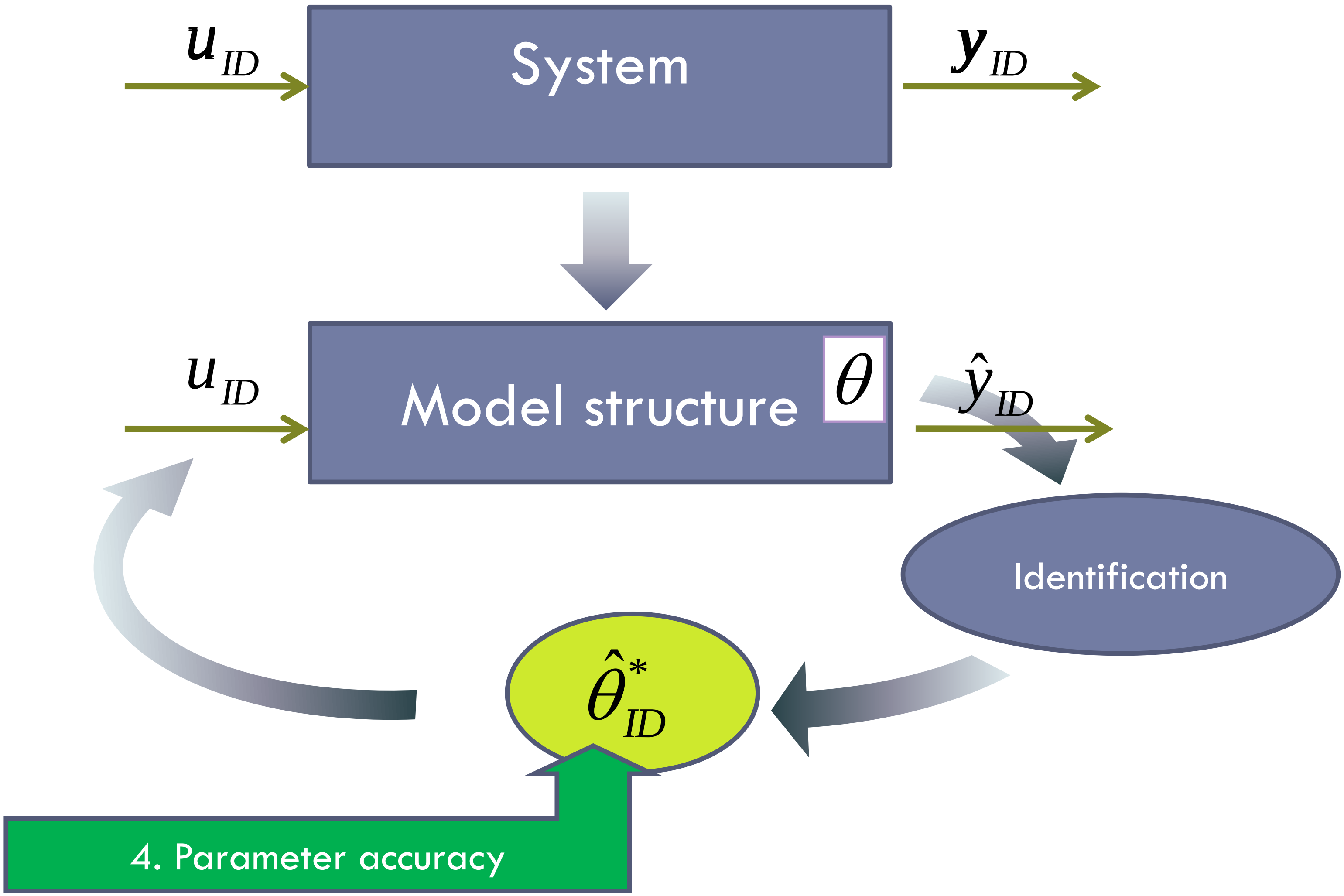
Online system identification example

- Online: results

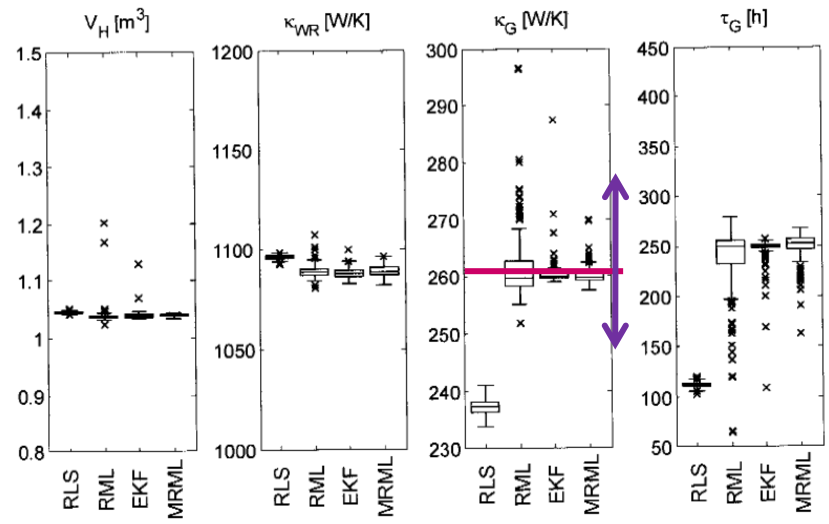


Source: Bianchi 2006

Parameter estimation



Parametric uncertainty



- Estimator properties
  - Unbiased estimator

$$E\left(\hat{\theta}\right) = \theta^*$$

  - Efficient estimator

$$F^{-1}\left(\hat{\theta}\right) \leq C\left(\hat{\theta}\right) \Rightarrow F^{-1}\left(\hat{\theta}\right) \approx C\left(\hat{\theta}\right)$$

    - With the Variance-Covariance matrix → Confidence region!

$$C\left(\hat{\theta}\right) = E\left(\left(\hat{\theta} - \theta^*\right)\left(\hat{\theta} - \theta^*\right)^T\right)$$

As small as possible!

    - And the Fisher Information matrix

$$F\left(\hat{\theta}\right) = \frac{1}{\sigma^2} \sum_{i=1}^{n_m} \left[\frac{\partial y_{(x_i,\theta)}}{\partial \theta}\right]_{\theta=\hat{\theta}} \left[\frac{\partial y_{(x_i,\theta)}}{\partial \theta}\right]_{\theta=\hat{\theta}}^T$$

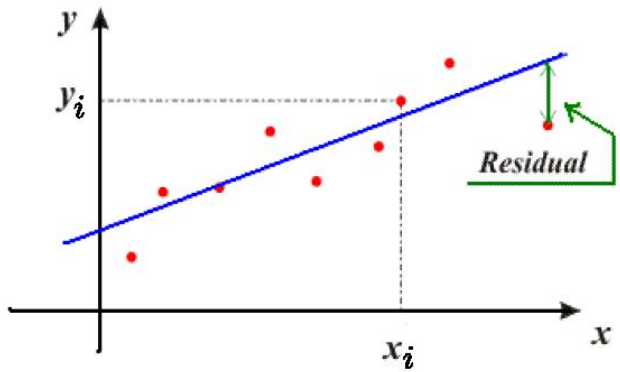
As large as possible!

  - Gaussian distributed

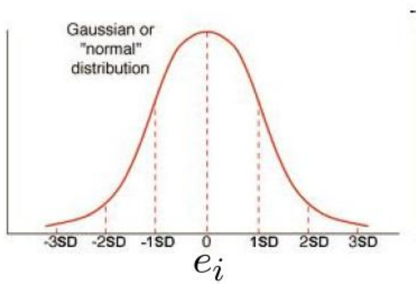
$$\hat{\theta} =\sim N\left(\theta^*, F^{-1}\left(\theta^*\right)\right)$$

Parametric uncertainty

- Estimator properties
  - If additive, independent, Gaussian distributed noise @ output



$$e_i = r_i = y_i - f(x_i, p)$$



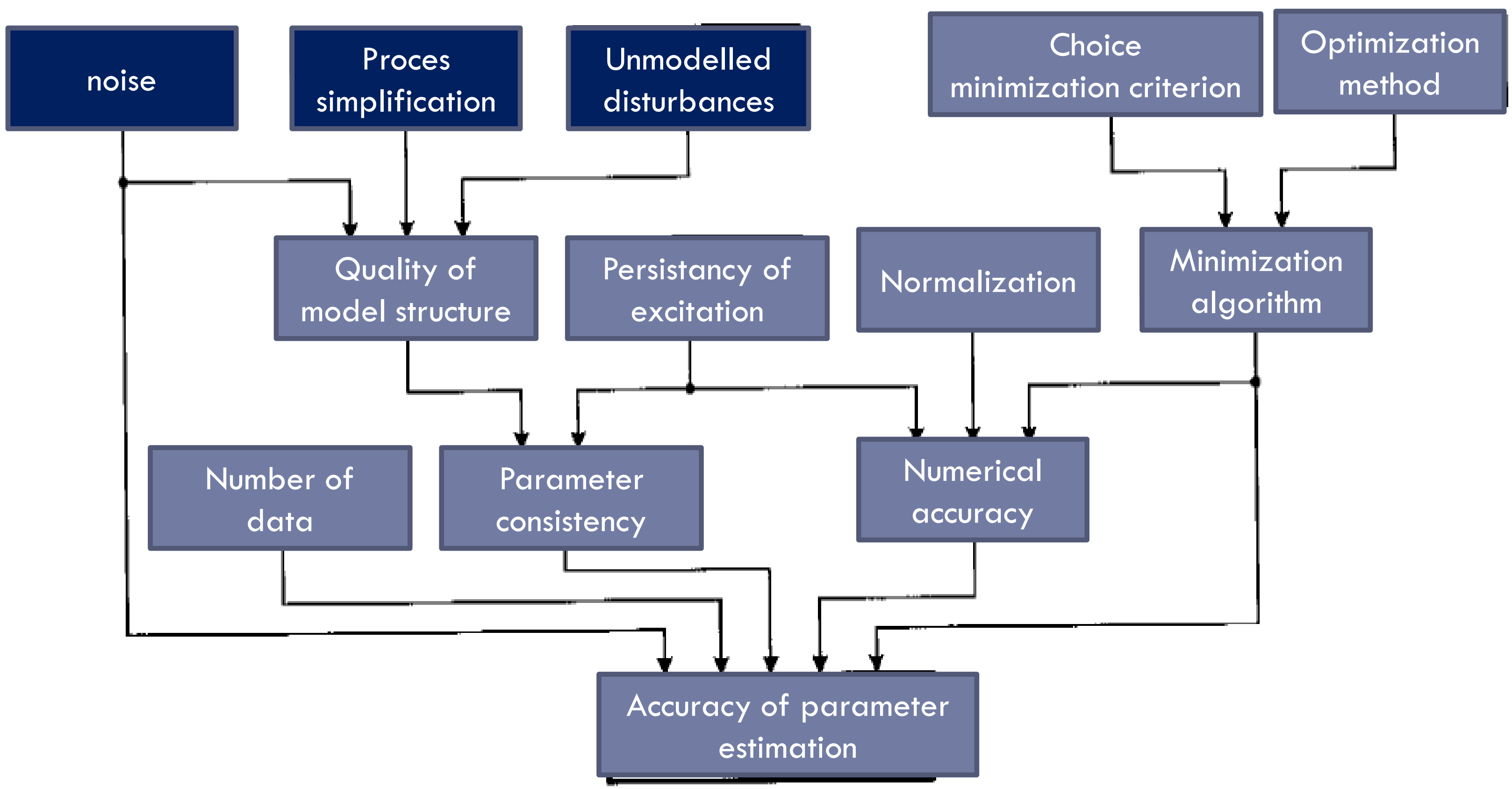
  - And for a large number of measurement data
  - Then the Fisher Information matrix can be approximated by:

$$F(\hat{\theta}) \approx \frac{1}{MSE} J^T J$$

    - With the mean squared error $MSE = \dfrac{e^T e}{n_m - n_\theta}$
    - And $J^T J$ the Hessian approximation of the cost function at $\hat{\theta}$

Parameter uncertainty

- Factors determining parameter estimation accuracy



Source: Bianchi 2006

Parameter uncertainty

- Quality of parameter estimation depends on:
  - Measured variables: MISO versus MIMO
  - Measurement quality: measurement noise variance
  - Amount of disturbances: input noise variance

Given

  - Model order: 3rd versus 2nd order
  - Parameter estimation method
    - ARX → Least Squares (LS)
    - Output Error (OE) → Prediction Error Method (PEM)
    - State Prediction Error with Kalman filter (KF) → PEM
  - Excitation signal

To be selected

Online system identification example

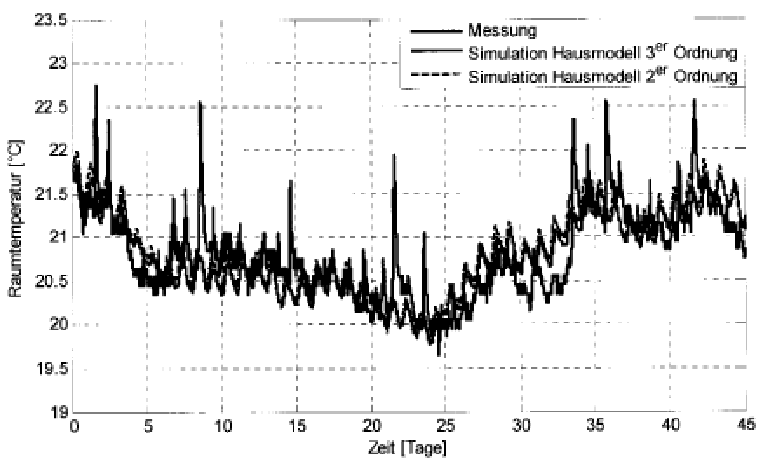
- 5. Model validation
  - Parameter values

	Hausmodell 3er Ord.	Hausmodell 2er Ord.
V_H	1.28 [m^3]	1 [m^3]
κ_{WB} bzw. κ_{WR}	1160 [W/K]	1100 [W/K]
$m_B c_B$	455.10^5 [J/K]	-
κ_B	6155 [W/K]	-
κ_G	260 [W/K]	260 [W/K]
τ_G	240 [h]	240 [h]
ΔT_{Stat}	3.3 [K]	3.3 [K]

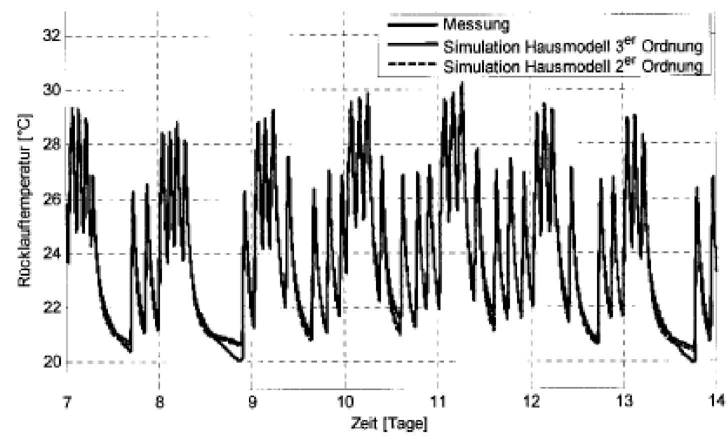
Source: Bianchi 2006

Online system identification example

- 5. Model validation
  - Time domain



Comparison of measured zone temperature with simulation results from 2nd and 3rd order model.



Comparison of measured return water temperature with simulation results from 2nd and 3rd order model.

- Both: Mean zone temperature correctly predicted
- Both: Variations due to solar radiation not predicted

Source: Bianchi 2006

Online system identification example

- 6. Evaluation control performance
  - Objectives
    - Thermal comfort & cost efficiency

      → prediction hourly Q_{dem} required… with available sensors
  - In case of floor heating systems
    - Q_{dem} depends on difference mean zone temperature T_{zone} and mean ambient air temperature T_{amb}
    - Fast variations for T_{zone} due to solar radiation can not be cancelled out due to inherent inertia of floor heating system
  - Conclusion
    - Both models fit purpose as hourly Q_{dem} is accurately predicted!
    - 2^{nd} order model better suited for online identification

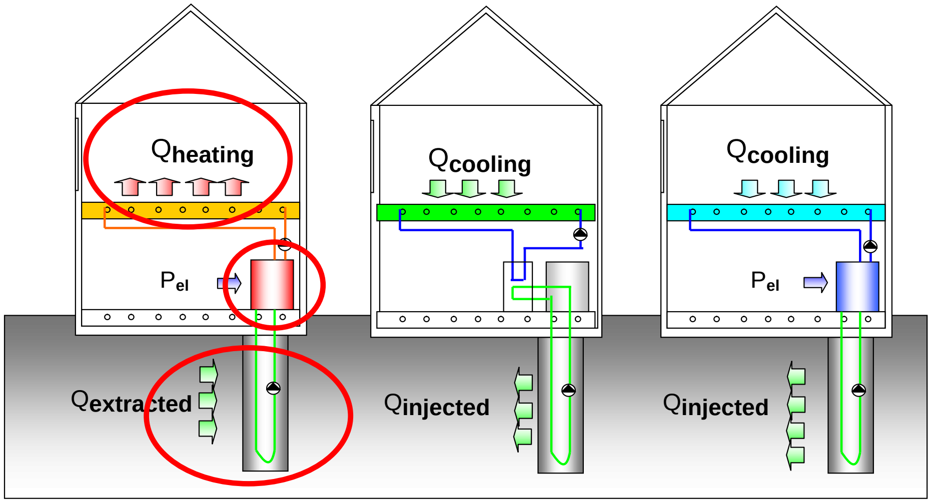
References

- Bianchi, M. (2006). **Adaptive Modellbasierte Prädiktive Regelung einer Kleinwärmepumpenanlage**, Institut für Mess- und Regeltechnik, ETH Zürich, p. 235.
- Parameter and State Estimation course, OPTEC, K.U.Leuven, 30-31 augustus 2010, www.kuleuven.be/optec

Applications in building control

- **Applications in building control**
  - Heating curve control
  - MPC for heavy-weight solar building
  - MPC for heat pump system with floor heating
  - MPC for ground coupled heat pump system
  - MPC for multizone building

Ground coupled heat pump example



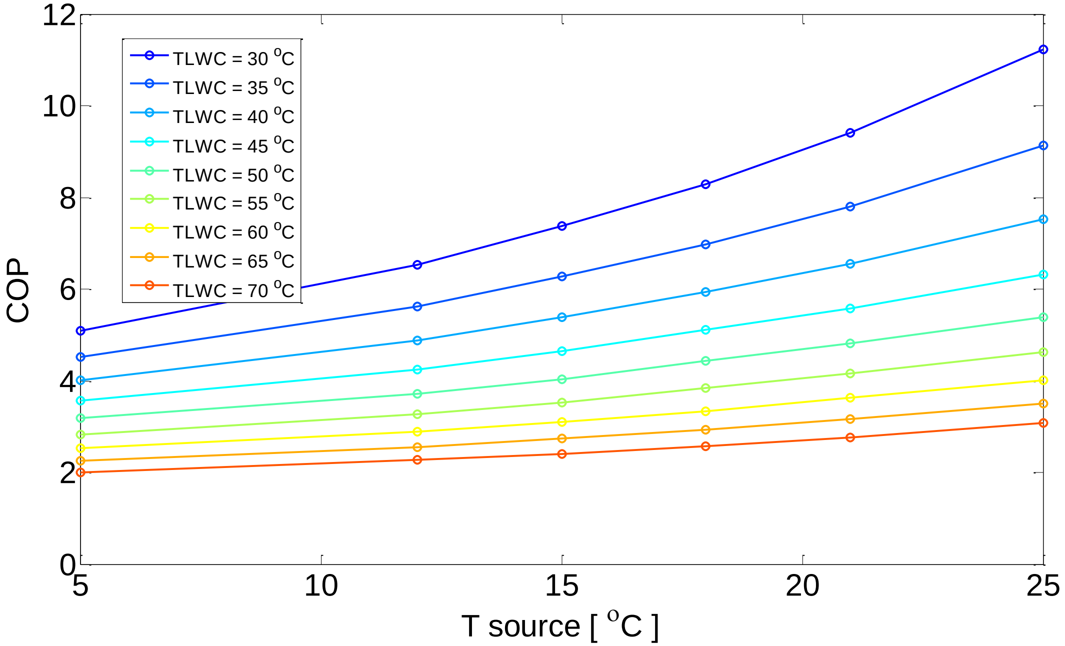
- 1. Control objectives
  - Thermal comfort at building side
  - Minimal electricity cost
  - Thermal balance at borefield side

Ground coupled heat pump example

- 1. Control objective
  - A. Minimal electricity cost

$$COP = \frac{\dot{Q}_{supply}}{P_{compressor}}$$

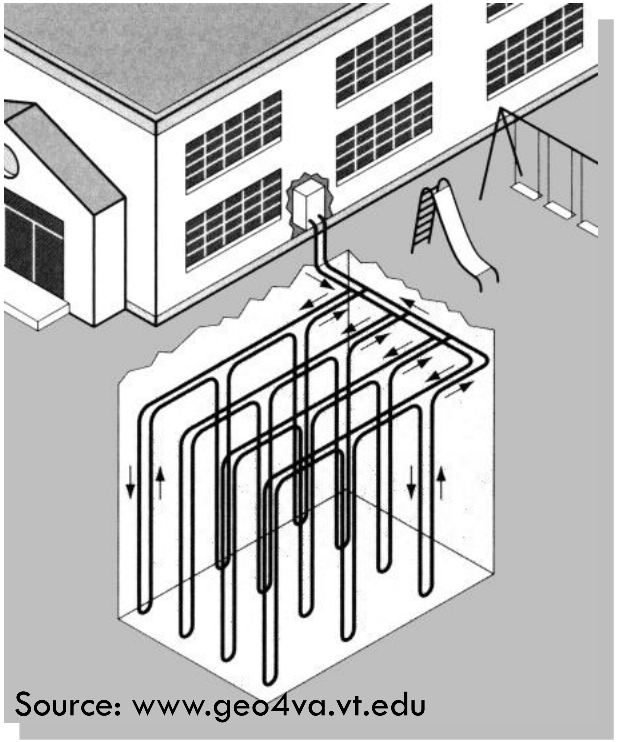
$$COP_{\max} = \frac{T_{supply}}{T_{supply} - T_{source}}$$

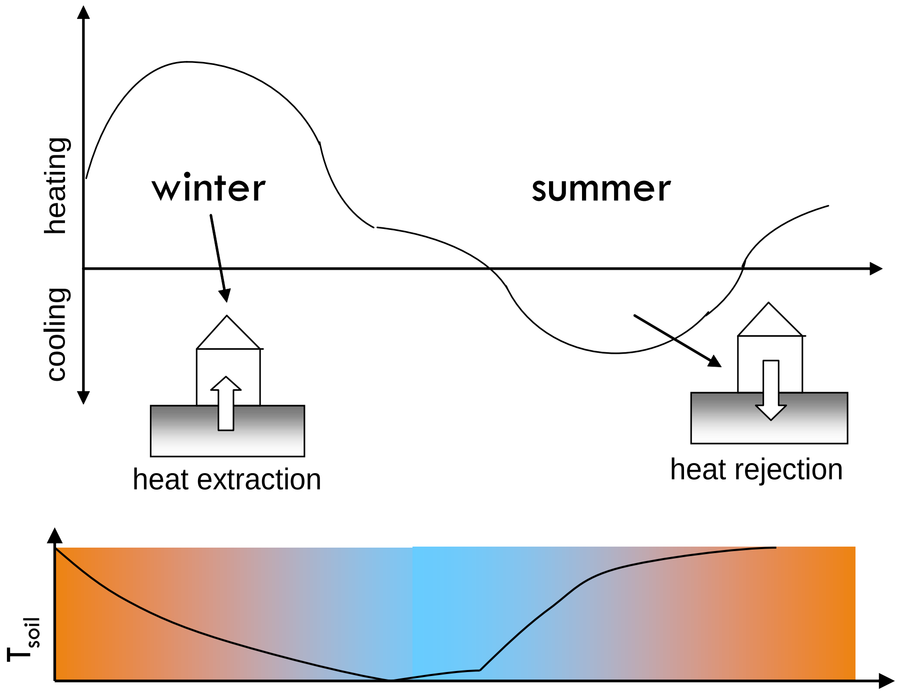


→ Brine water temperature needed for short-term control

Ground coupled heat pump example

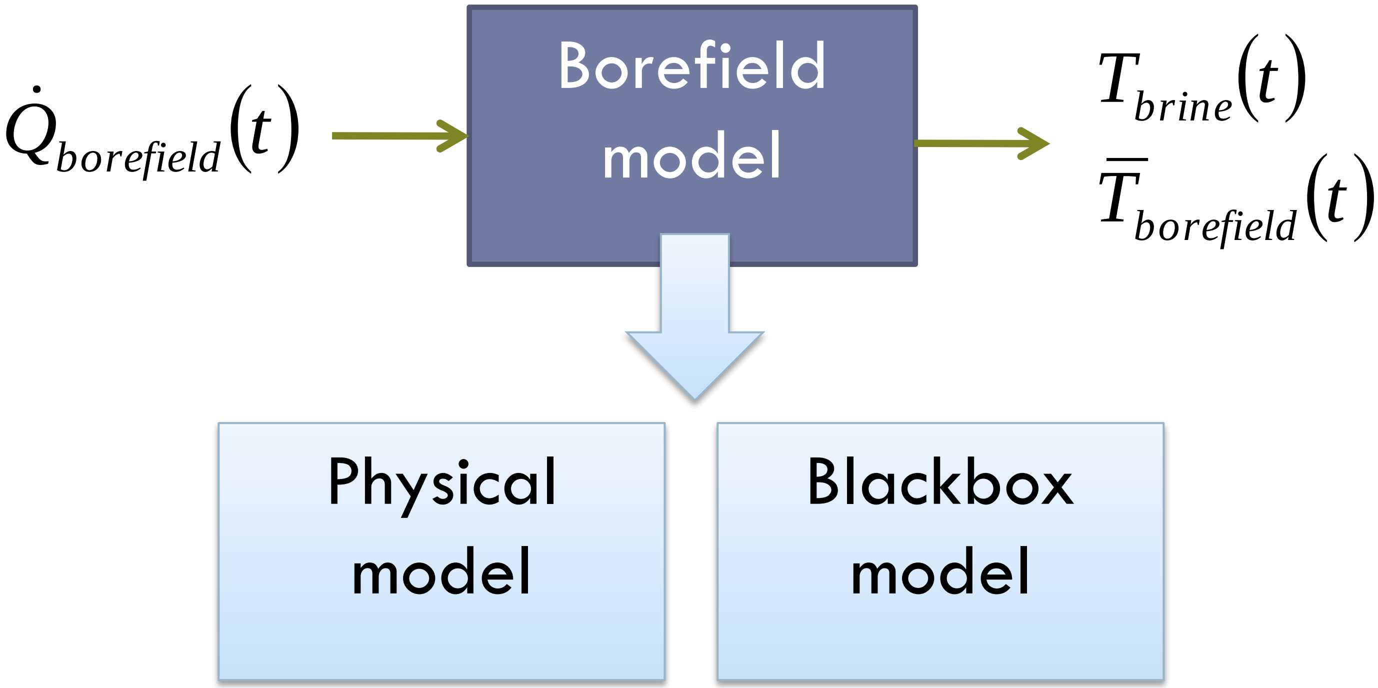
- 1. Control objectives
  - B. Thermal balance at borefield side





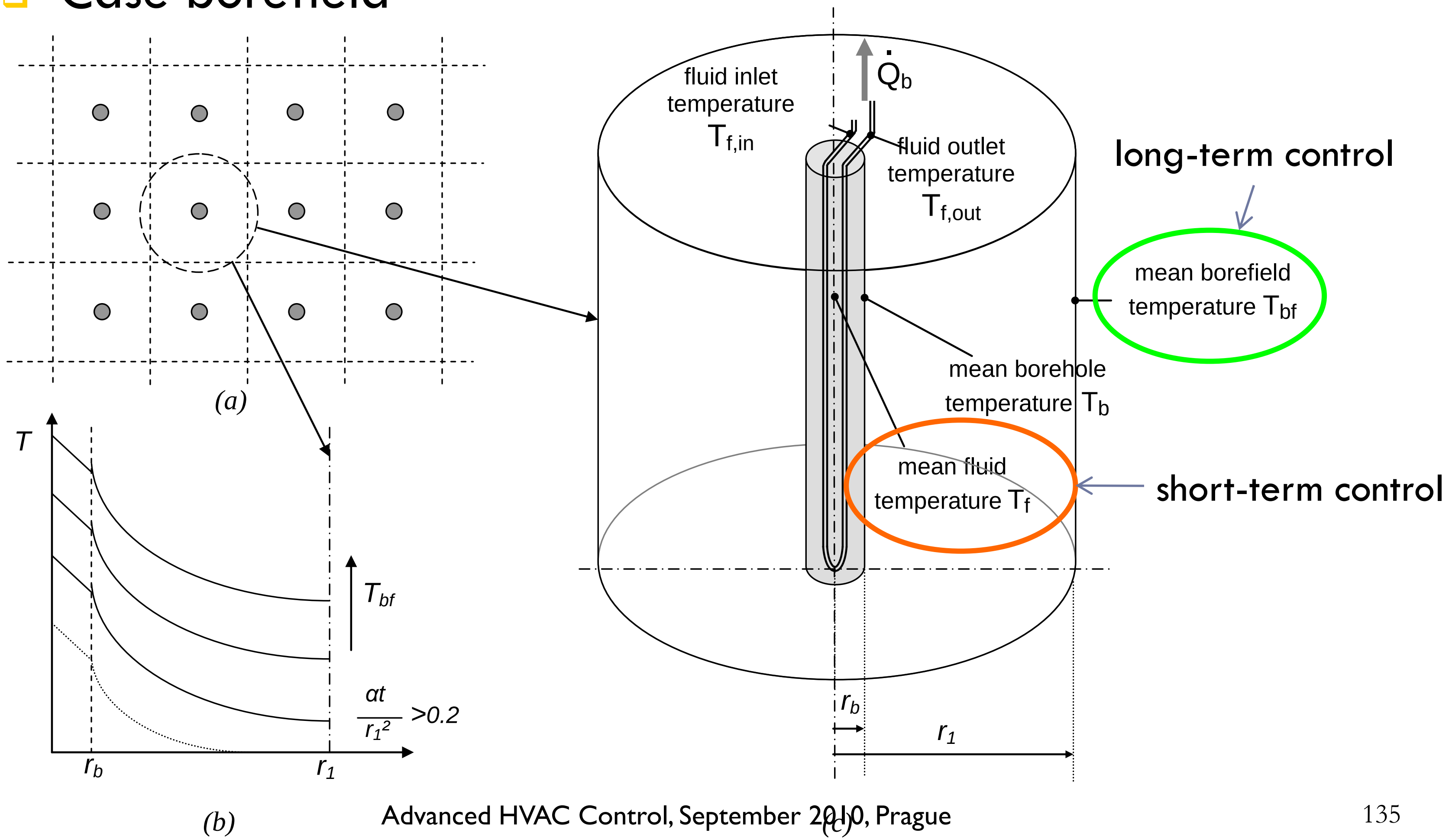
→ Measure for stored energy needed for long-term control

- 2. Model input and outputs



Ground coupled heat pump example

- 2. Physical model
  - Case borefield

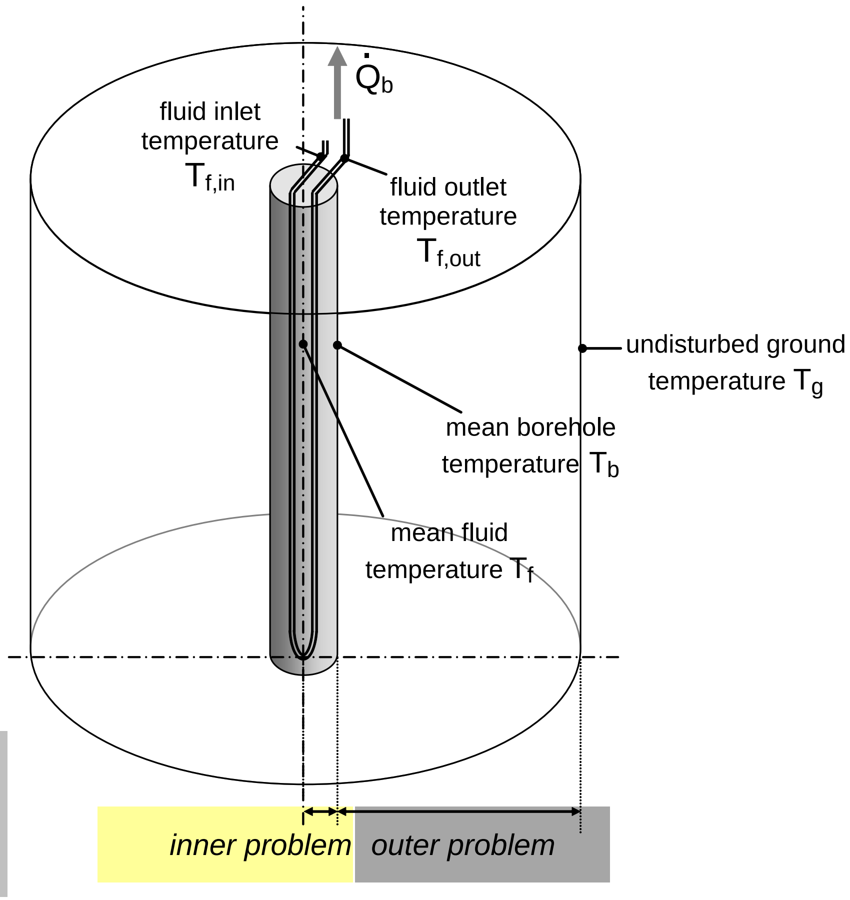


Ground coupled heat pump example

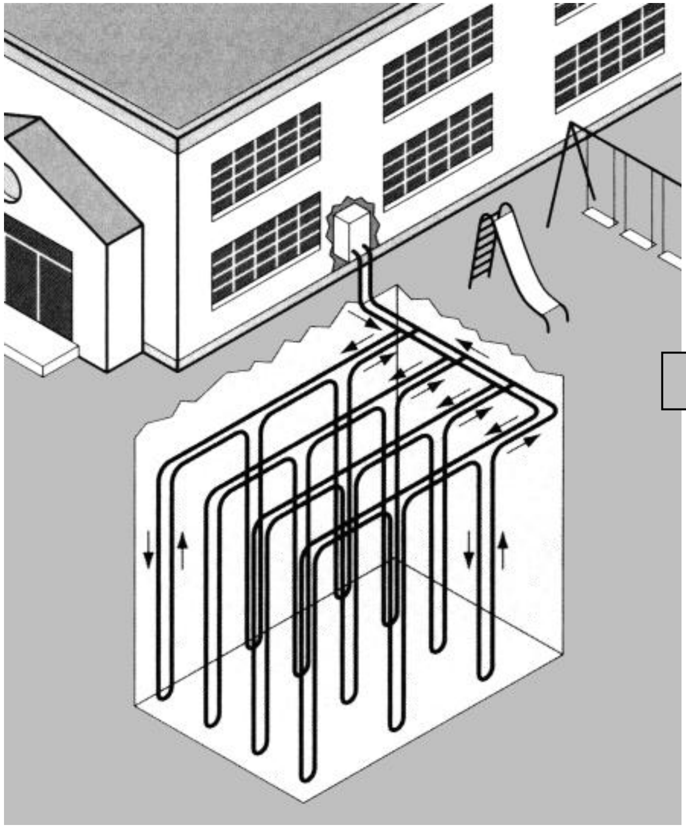
- 2. Physical model
  - Heat transfer processes
    - Between brine and ground: convection + conduction
    - In the ground: conduction

$$Q_b = \frac{T_b - T_f}{R_b}$$

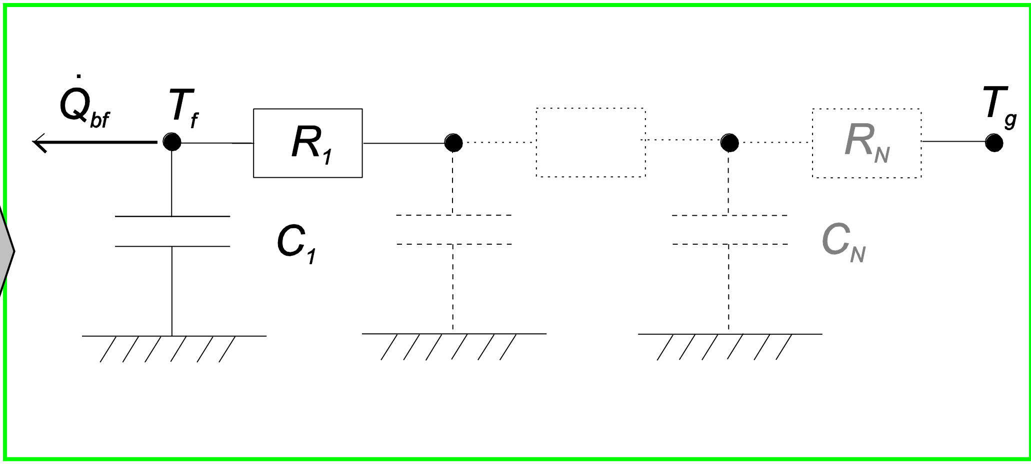
$$T_f = T_g - \frac{Q_b}{4\pi k_g H}\left(\ln(t) + \ln\left(\frac{4\alpha}{r_b^{\ 2}}\right)\right) - \frac{Q_b}{H} R_b$$



Ground coupled heat pump example

- 

- 3. Model structure

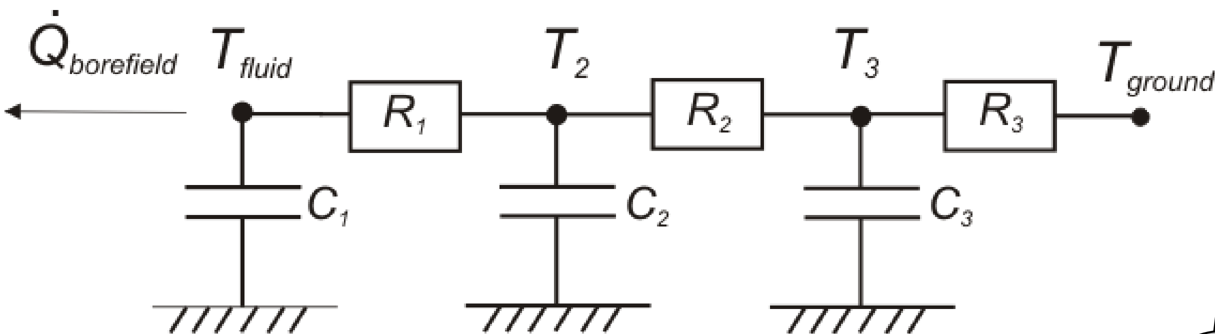


Grey-box model

TRNSYS type557b

Matlab

- 3. Model structure
  - RC-network representation



  - State space formulation

$$\boxed{\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx\end{aligned}}$$

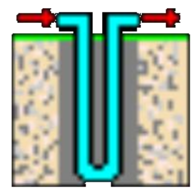
$$\begin{pmatrix}\dot{T}_1\\ \dot{T}_2\\ \dot{T}_3\end{pmatrix} = \begin{pmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} & 0\\ \frac{1}{R_1C_2} & -\left(\frac{1}{R_1C_2}+\frac{1}{R_2C_2}\right) & \frac{1}{R_2C_2}\\ 0 & \frac{1}{R_2C_3} & -\left(\frac{1}{R_2C_3}+\frac{1}{R_3C_3}\right)\end{pmatrix}\begin{pmatrix}T_1\\ T_2\\ T_3\end{pmatrix} + \begin{pmatrix} -\frac{1}{C_1} & 0\\ 0 & 0\\ 0 & \frac{1}{R_3C_3}\end{pmatrix}\begin{pmatrix}\dot{Q}_{bf} & T_g\end{pmatrix}$$

$$T_f = \begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}T_1\\ T_2\\ T_3\end{pmatrix}$$

- 4. Parameter estimation
  - Identification data

input variables →

$\dot{m},\ T_{f,inlet}\ \ldots$

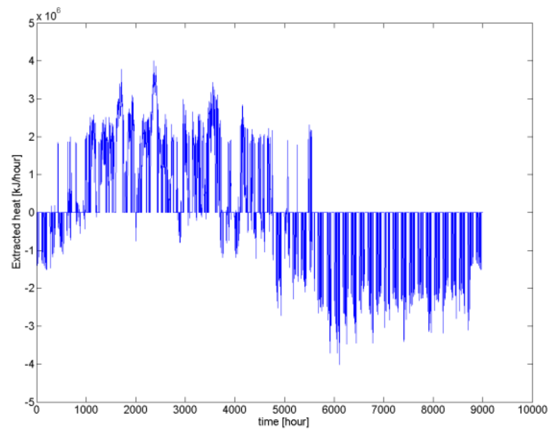


→ output variables

$\dot{Q}_b,\ T_{f,outlet},\ T_{bf}$

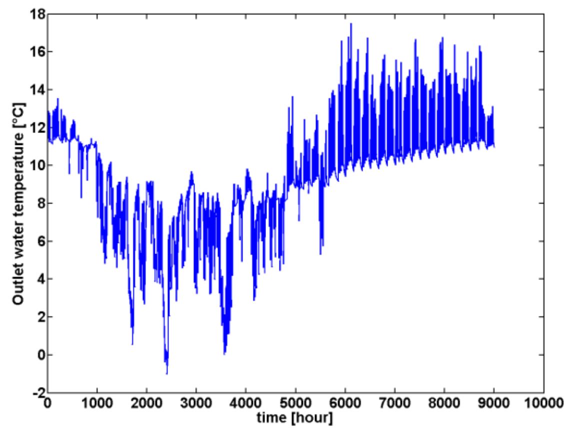
TRNSYS Type557b

RC-model input



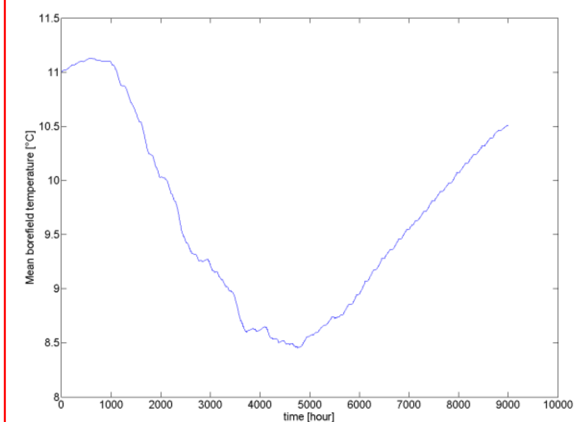
Heat extraction from borefield [kJ/hour]

RC-model output 1



Mean fluid temperature [°C]

RC-model output 2



Mean borefield temperature [°C]

Ground coupled heat pump example

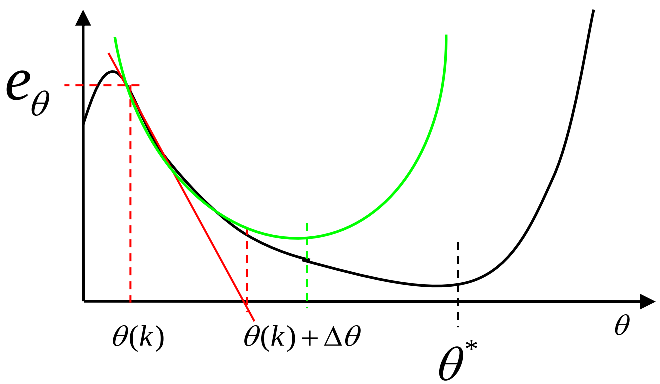
- 4. Parameter estimation
  - Model with *2N* parameters
  - Prediction Error Method & least squares
    - *m* measurements $y(t)$
    - *m* model outputs $y_{mod}(t)$
    - *m* model errors $e(t)$

$$\theta^* = \arg\min_{\theta} \frac{1}{2} e^T e \quad \text{with} \quad e = \begin{pmatrix} y_{\text{mod}}(t_0) - y(t_0) \\ y_{\text{mod}}(t_1) - y(t_1) \\ \vdots \\ y_{\text{mod}}(t_m) - y(t_m) \end{pmatrix}$$

  - → Nonlinear optimization problem
    - Solved with gradient-based method
    - Importance of choice initial value

Gradient-based methods

- Principle



$$J = \begin{pmatrix} \frac{\partial y(t_1)}{\partial \theta_1} & \frac{\partial y(t_1)}{\partial \theta_2} & \cdots & \frac{\partial y(t_1)}{\partial \theta_{Np}} \\ \frac{\partial y(t_2)}{\partial \theta_1} & \frac{\partial y(t_2)}{\partial \theta_2} & \cdots & \frac{\partial y(t_2)}{\partial \theta_{Np}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial y(t_{Nm})}{\partial \theta_1} & \frac{\partial y(t_{Nm})}{\partial \theta_2} & \cdots & \frac{\partial y(t_{Nm})}{\partial \theta_{Np}} \end{pmatrix}$$

Jacobian matrix

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \Delta\theta$$

- Steepest gradient $\Delta\theta = -J\ \mathrm{e}$
- Gauss-Newton $\Delta\theta = -\left(J^T J\right)^{-1} J\ \mathrm{e}$
- Levenberg-Marquardt $\Delta\theta = -\left(J^T J + \lambda\right)^{-1} J\ \mathrm{e}$

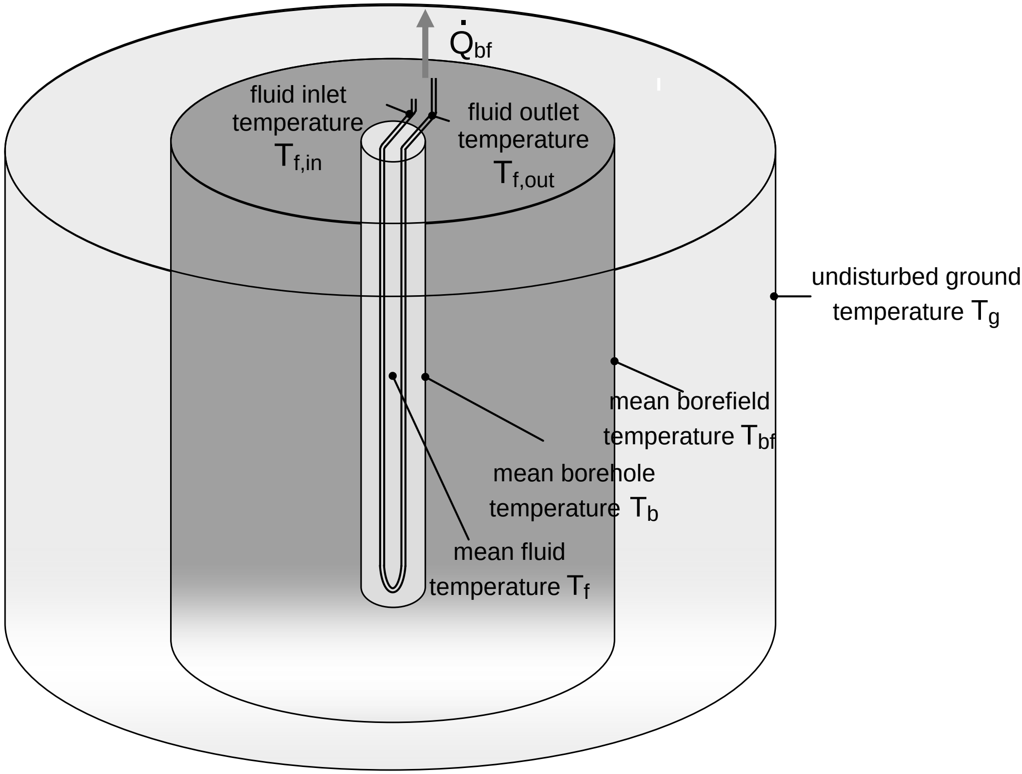
Ground coupled heat pump example

- 4. Parameter estimation
  - Initial values based on theory of heat conduction

$$\frac{\alpha t}{r^2} \approx 0.8$$

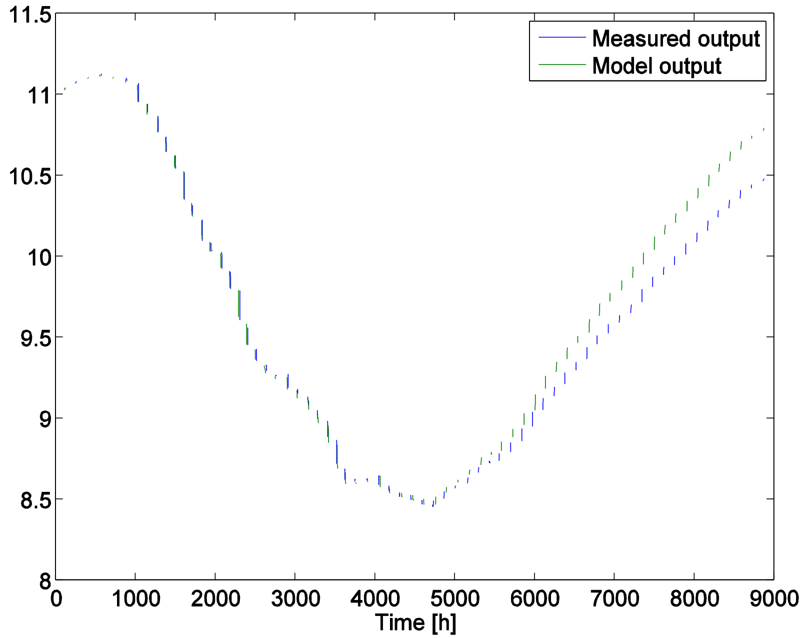
$$R = \frac{1}{2\pi k_g H} ln\left(\frac{r}{r_b}\right)$$

$$C = c_g \pi (r^2 - r_b{}^2) H$$

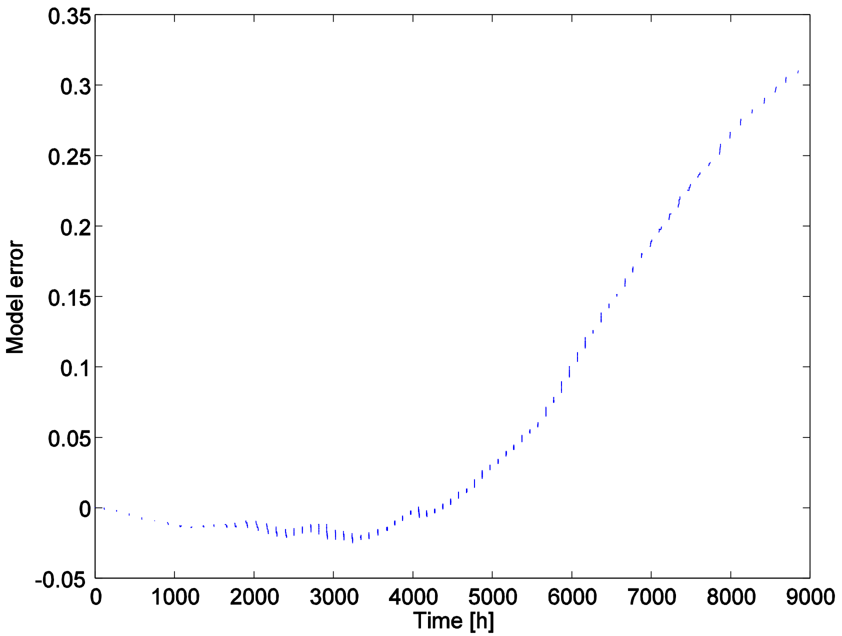


Ground coupled heat pump example

- 5. Model validation temperature T_{bf}



Mean borefield temperature

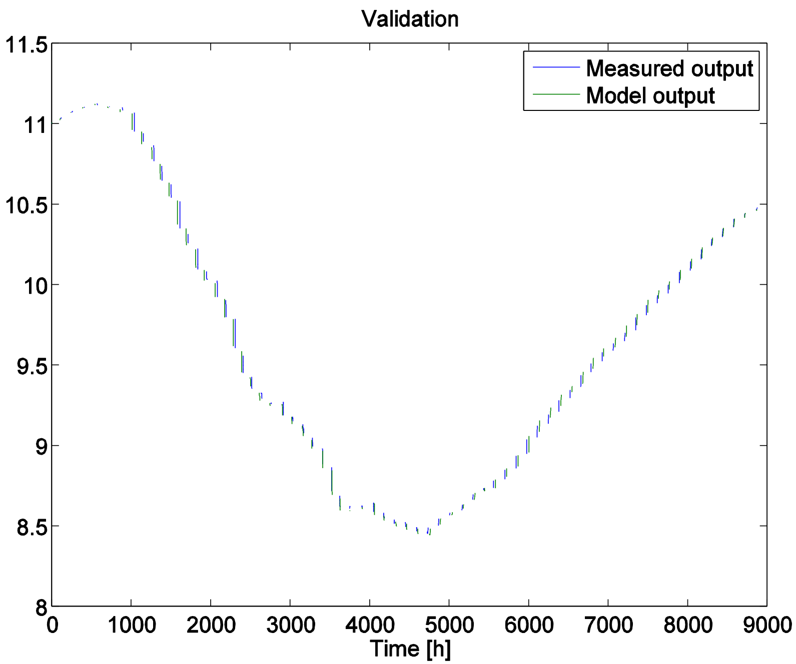


Error on mean borefield temperature

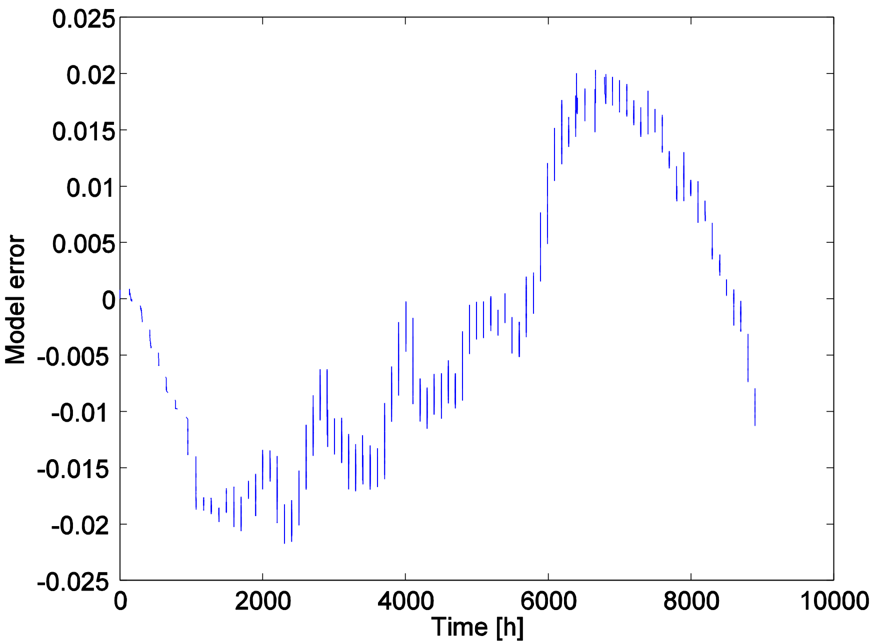
order = 1 & $time_{ID}$ = ½ year

Ground coupled heat pump example

- Evaluation models for mean borefield temperature



Mean borefield temperature

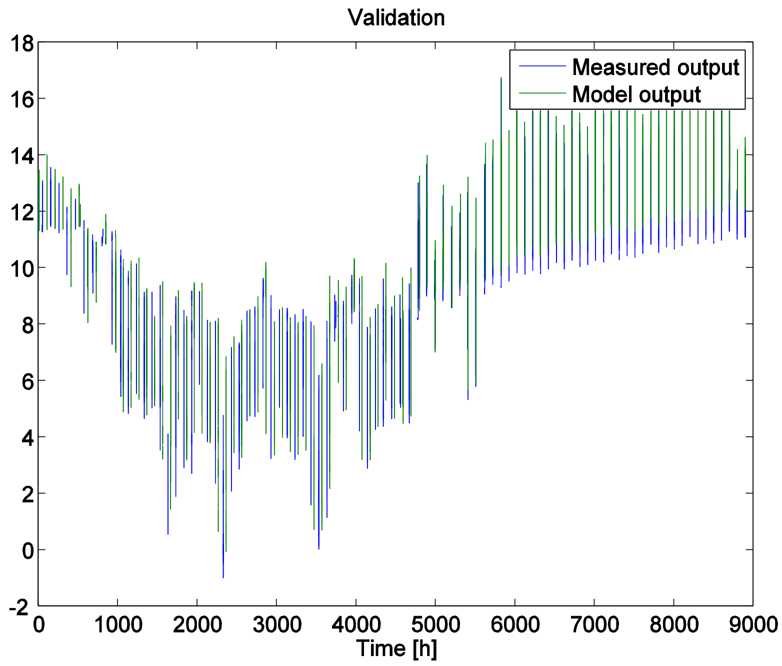


Error on mean borefield temperature

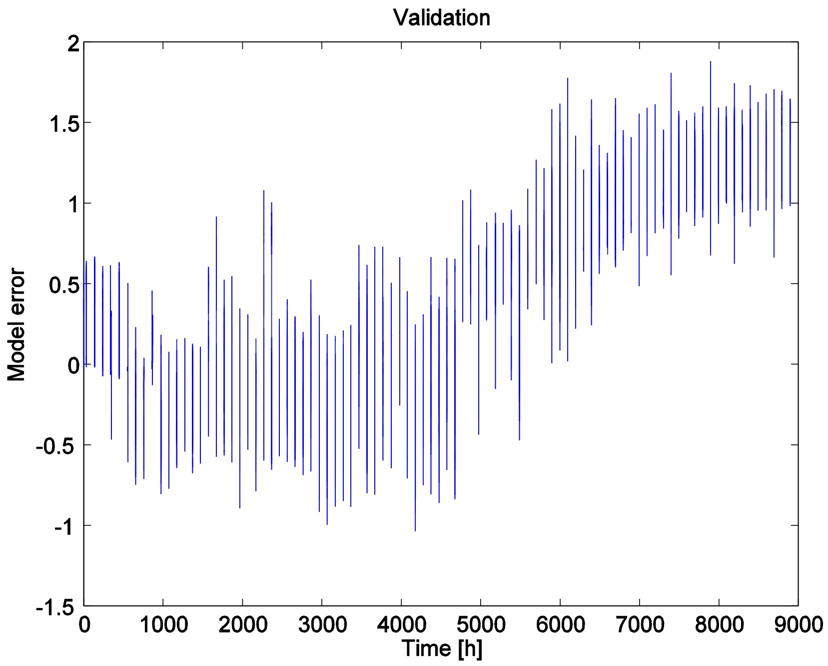
order = 2 & $time_{ID}$ = ½ year

Ground coupled heat pump example

- Evaluation models for mean fluid temperature T_f



Fluid temperature

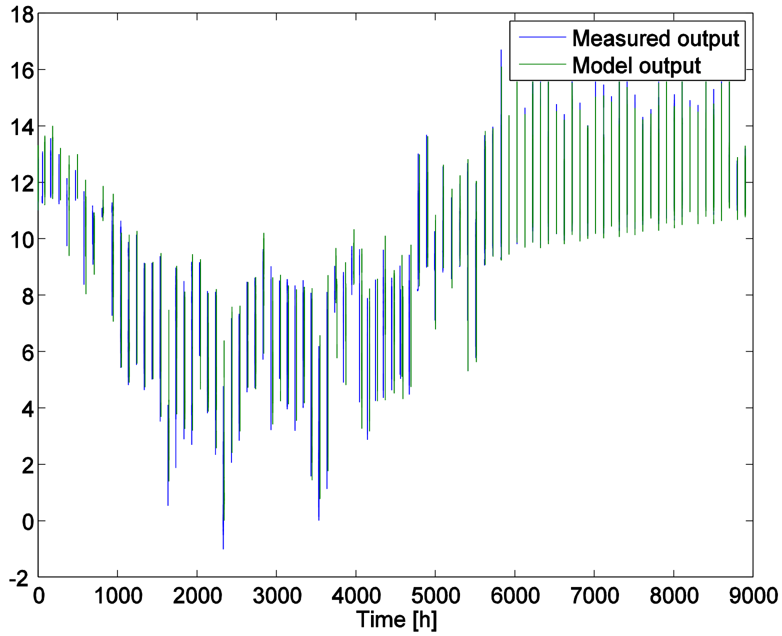


Error on fluid temperature

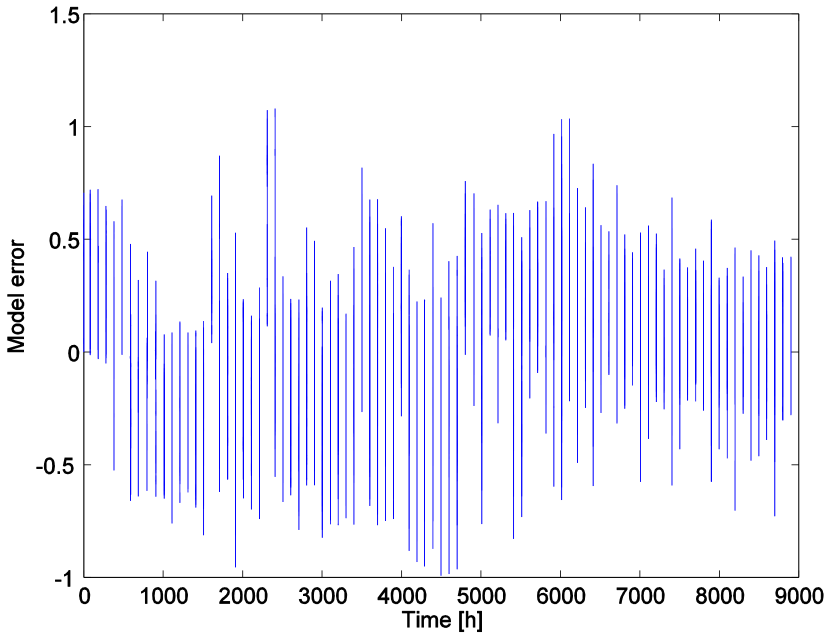
order = 4 & time_{id} = ½ year

Ground coupled heat pump example

- Evaluation models for mean fluid temperature T_f



Fluid temperature



Error on fluid temperature

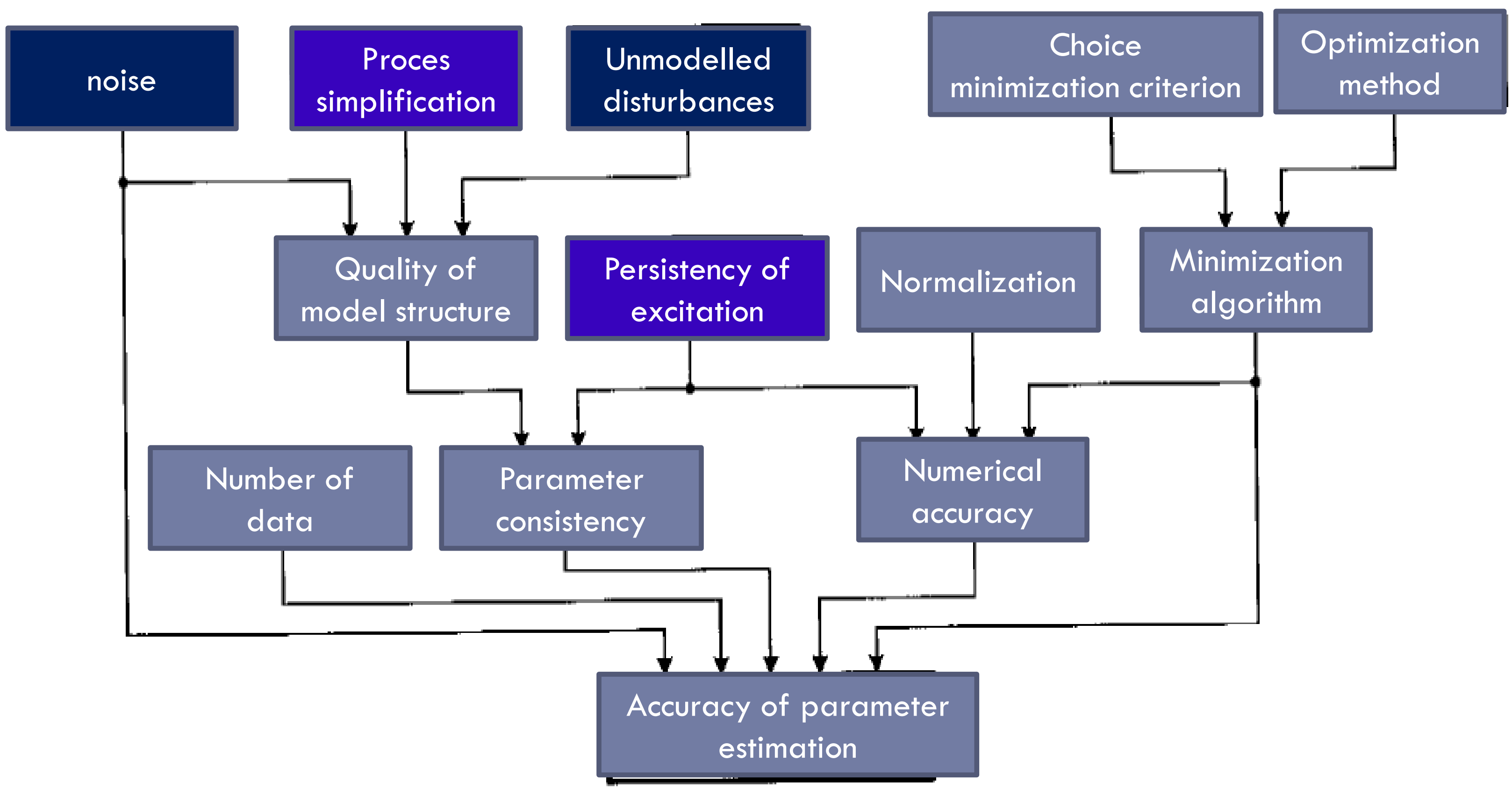
order = 5 & $time_{id}$ = ½ year

Ground coupled heat pump example

- Results
  - Optimal model order: to be determined
  - Problems with extrapolation
  - $\hat{\theta}^*$ depends on identification data set

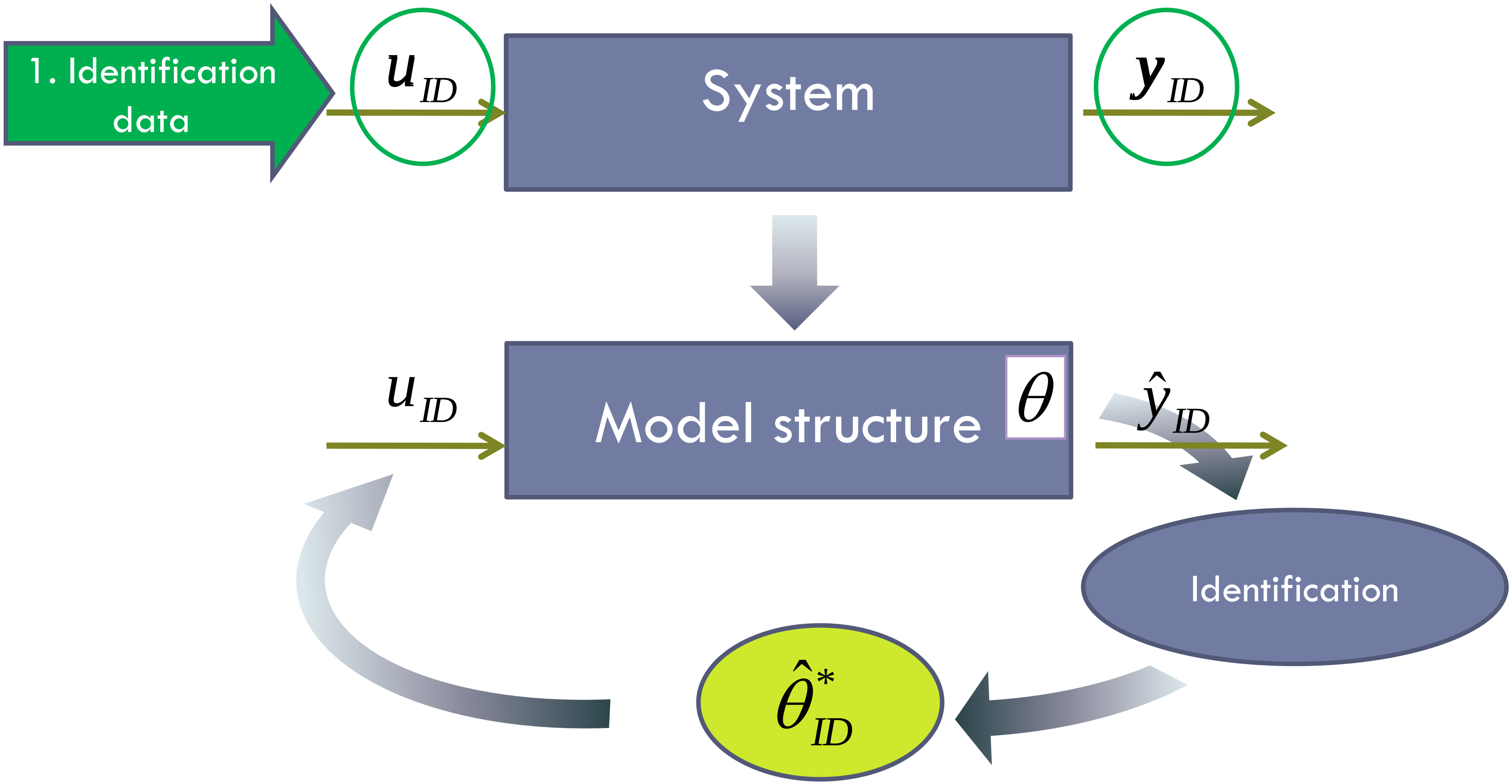
Parameter uncertainty

- Factors determining parameter estimation accuracy



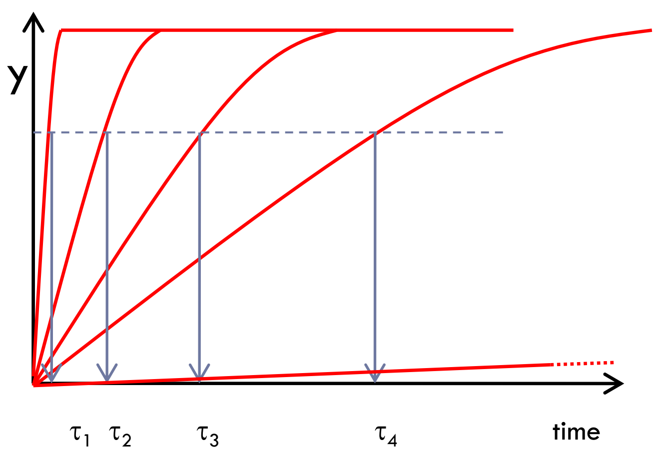
Source: Bianchi 2006

Choice of identification data



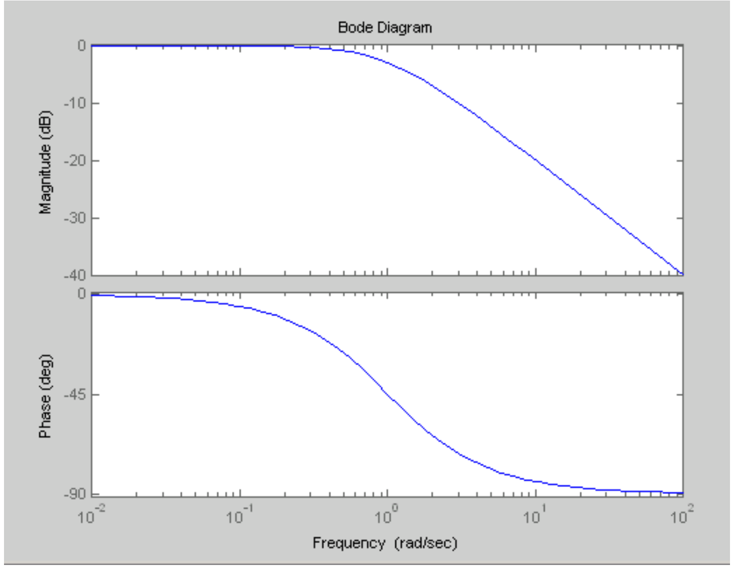
Choice of identification data

- Define dynamics of interest



Step response of system

System time constants



Bode plot of system

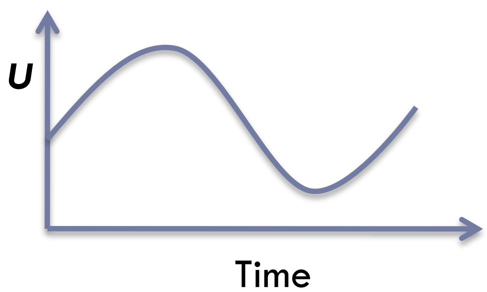
Frequency range of interest

$$\frac{1}{\tau_{\min}} = f_{sys,\max}$$

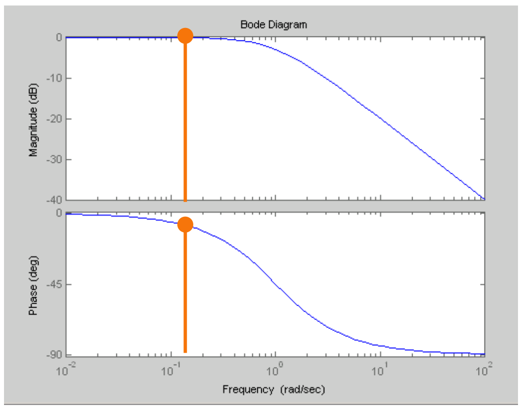
$$\frac{1}{\tau_{\max}} = f_{sys,\min}$$

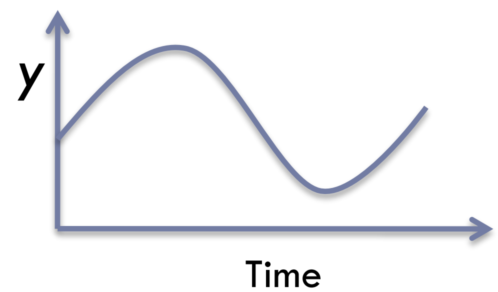
Choice of identification data

- Excite frequencies of interest



Sine input

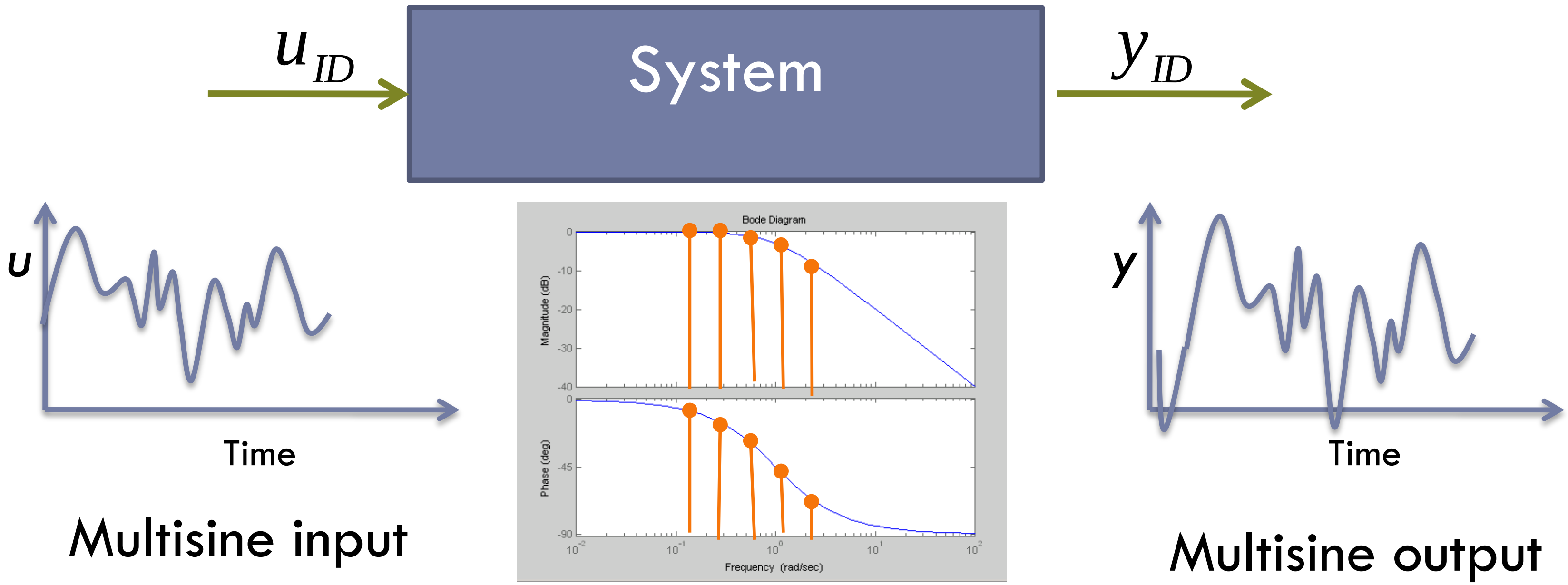




Sine output

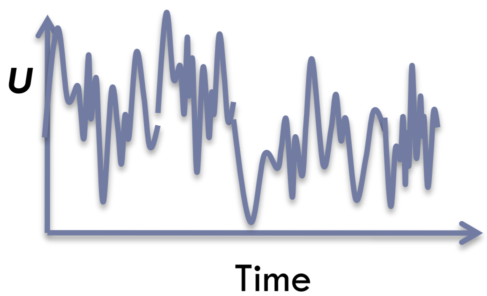
Choice of identification data

- Excite frequencies of interest

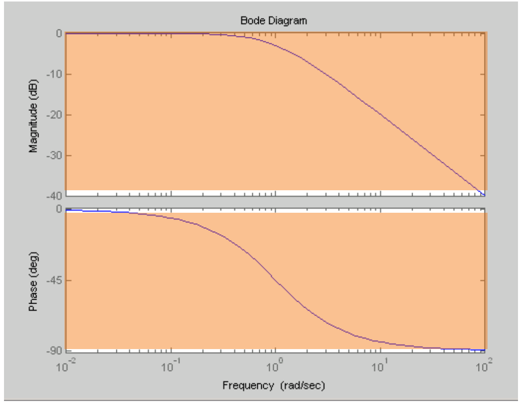


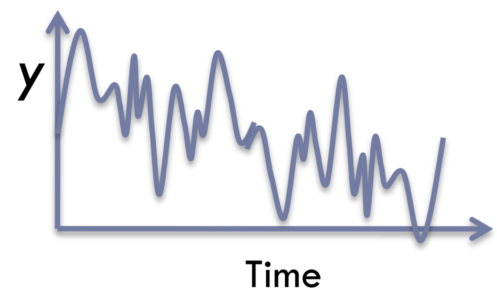
Choice of identification data

- Excite frequencies of interest



White noise input

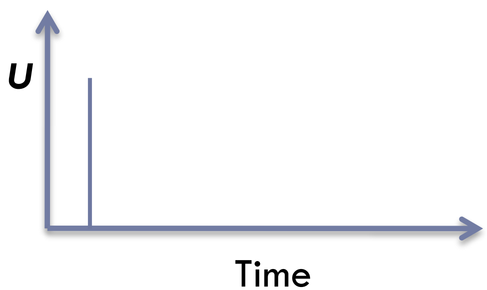




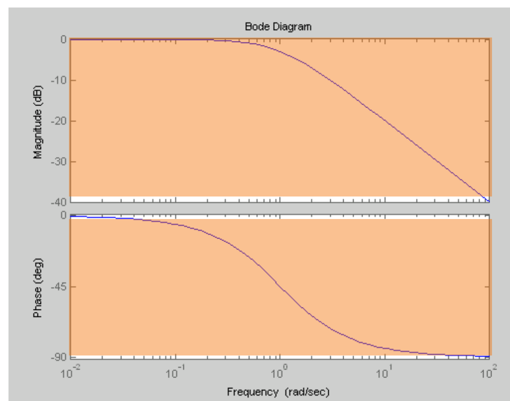
Coloured noise output

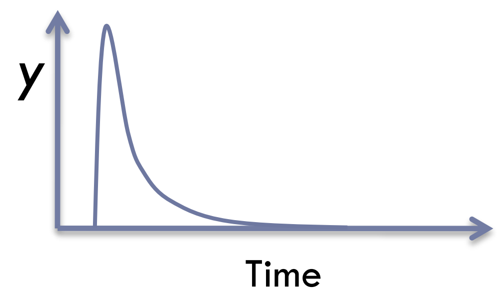
Choice of identification data

- Excite frequencies of interest



Impulse input

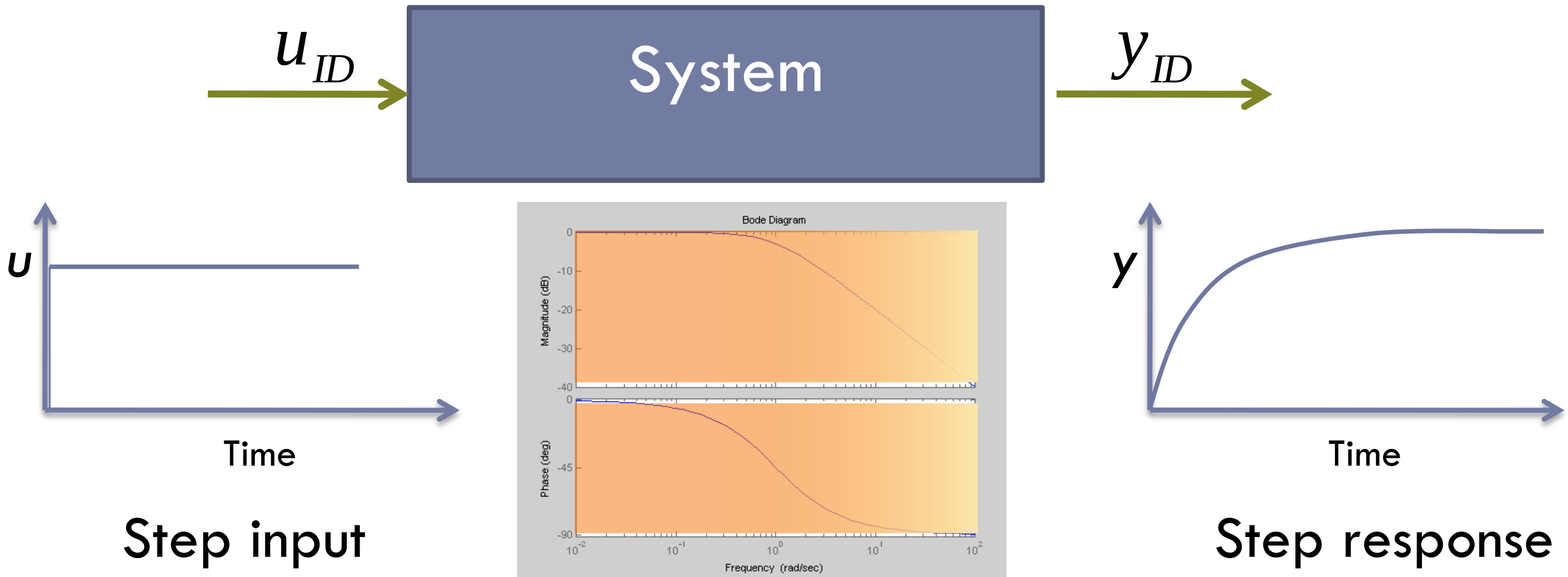




Impulse response

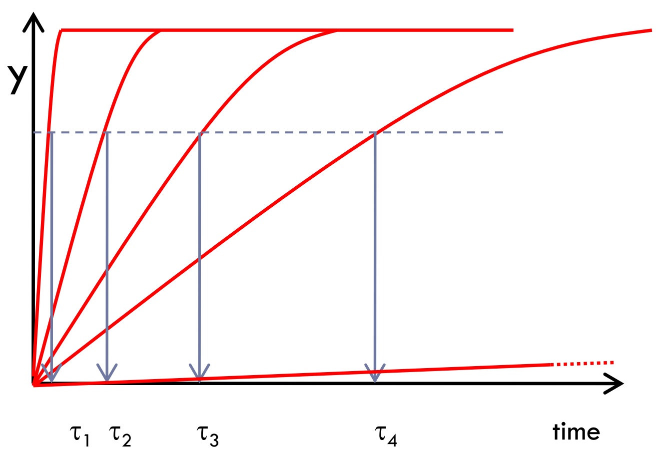
Choice of identification data

- Excite frequencies of interest

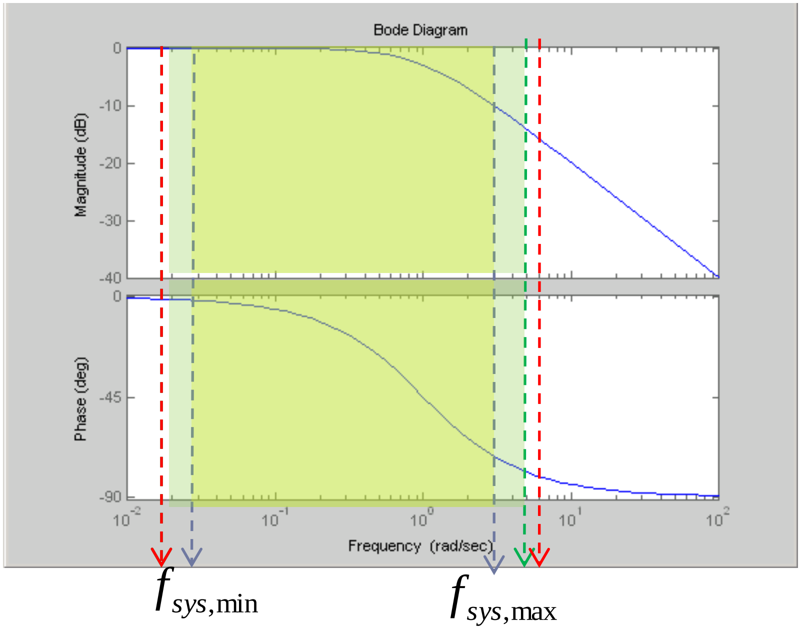


Choice of identification data

- 'Persitent excitation'



Step response of system



Bode plot of system

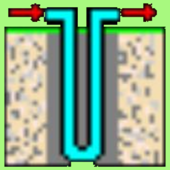
System	Excitation	Measurement
$\frac{1}{\tau_{\min}} = f_{sys,\max}$ →	$f_{ID,\max} \geq f_{sys,\max}$ →	$f_{sampling} \geq 2 f_{ID,\max} \rightarrow \Delta t_{sampling} \leq \frac{\tau_{\min}}{2}$
$\frac{1}{\tau_{\max}} = f_{sys,\min}$ →	$f_{ID,\min} \leq f_{sys,\min}$ →	$t_{ID,length} \geq \tau_{\max}$

Ground coupled heat pump example

- 4. Parameter estimation
  - … new set of identification data

Identification data

- Simulation

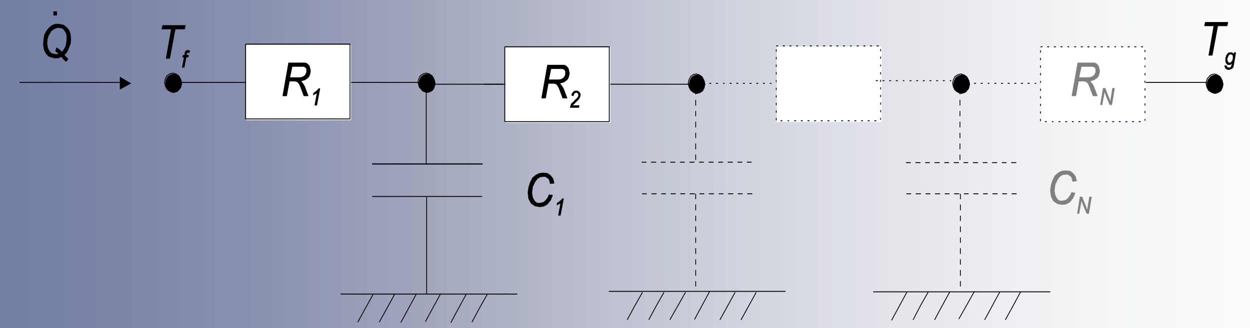
$\dot{m}, \dot{Q}$ →  → $T_{f,mean}$

multisine

•4 sets: period of 3 months, 6 months, 1 year, 10 year

Ground coupled heat pump example

- … and extra set of model structures



$$\frac{T'_f}{Q'} = \frac{b_z s^z + b_{z-1} s^{z-1} + \ldots + b_0}{a_p s^p + a_{p-1} s^{p-1} + \ldots + a_0}$$

$$g\left(t^*, \beta\right) = \frac{1}{2}\int_0^1 d\eta \int_0^1 \left[\frac{erfc(r^+ / 2\beta\sqrt{t^*})}{r^+} - \frac{erfc(r^- / 2\beta\sqrt{t^*})}{r^-} \right] \mathrm{d}\xi$$

Diffusion in semi-infinite medium

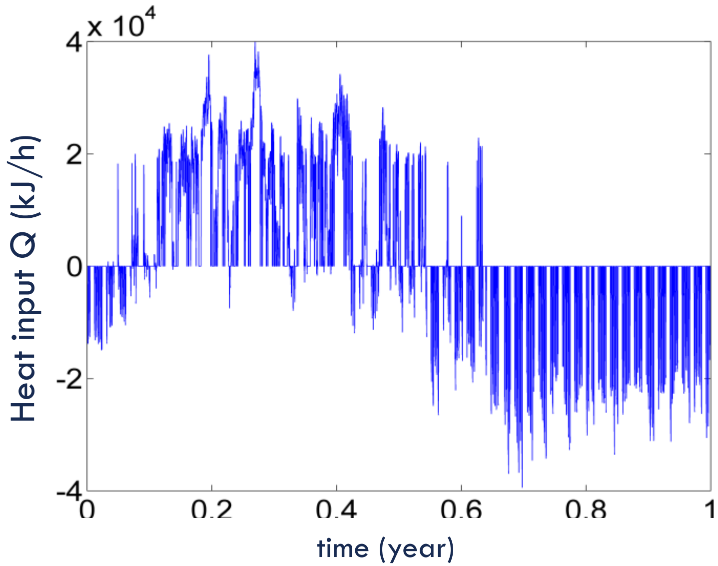
$$\frac{T'_f}{Q'} = \frac{b_z \left(\sqrt{s}\right)^z + b_{z-1}\left(\sqrt{s}\right)^{z-1} + \ldots + b_0}{a_p\left(\sqrt{s}\right)^p + a_{p-1}\left(\sqrt{s}\right)^{p-1} + \ldots + a_0}$$

Ground coupled heat pump example

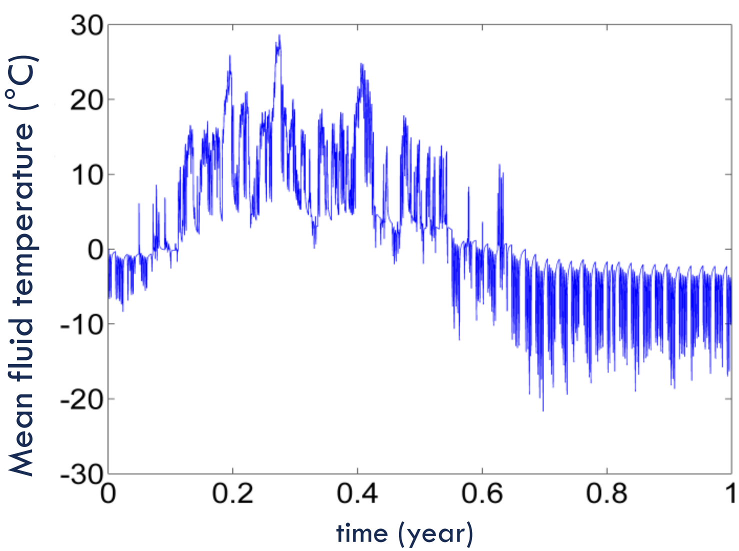
- 5. Model validation
  - Parameter values: physical?
  - Confidence interval parameters
  - Cross-validation in time domain
  - Modal analysis in frequency domain
  - Error analysis

Ground coupled heat pump example

- 5. Model validation
  - Time domain



input u_{val}

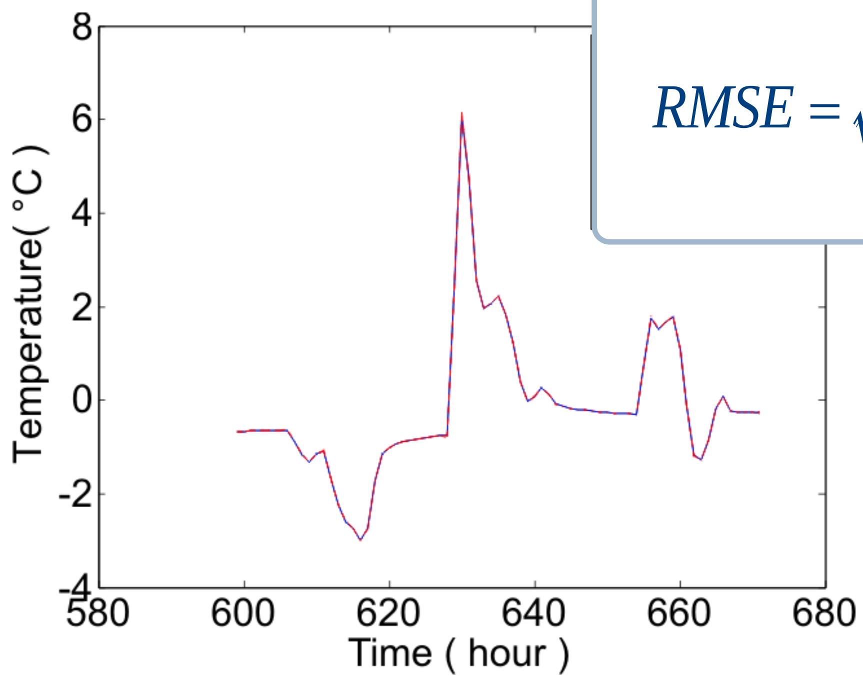


TRNSYS output y_{val}

Validation data set

Ground coupled heat pump example

- 5. Model validation
  - Time domain



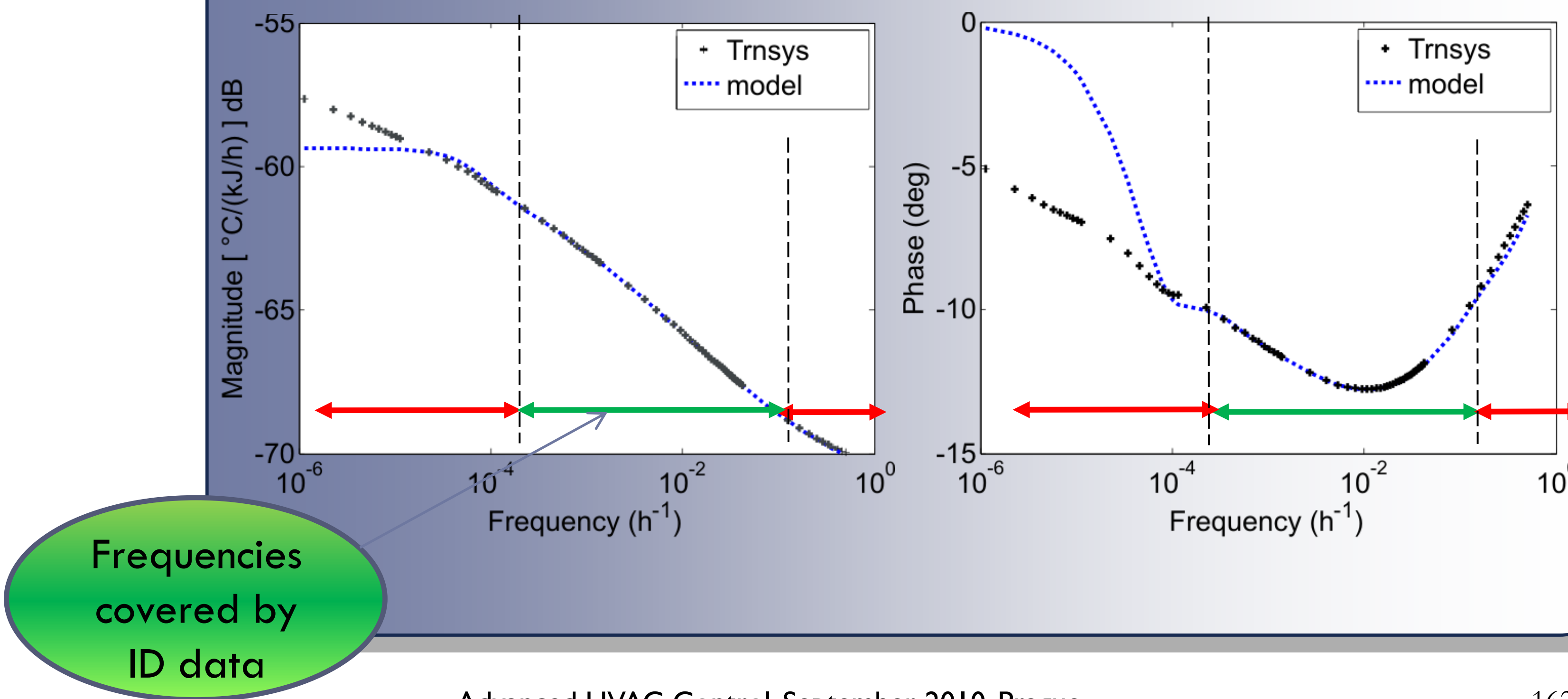
$$RMSE = \sqrt{\frac{\left(y - y_{mod}\right)^2}{N}}$$

Validation result RC-model (detail)

Ground coupled heat pump example

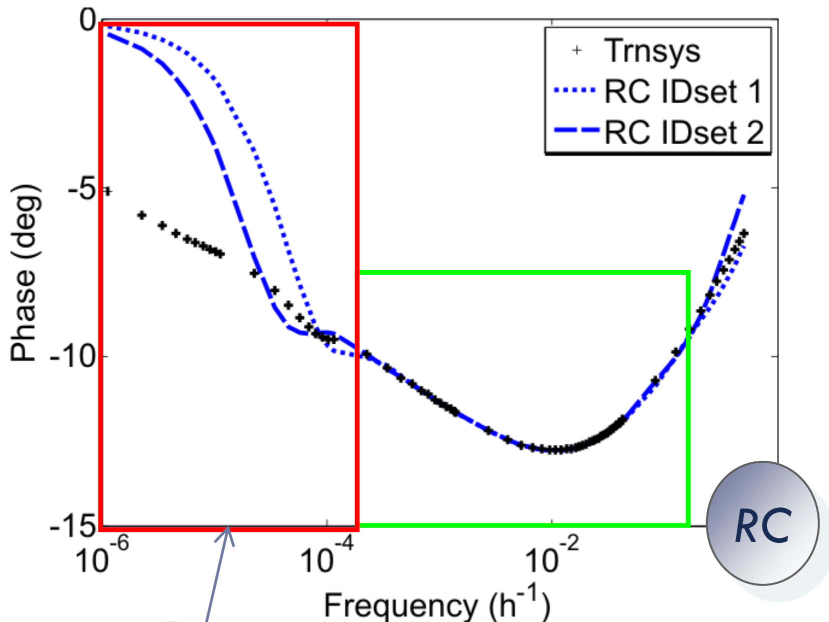
- 5. Model validation
  - Frequency domain

• Bode-plots RC-model/ TRNSYS-model

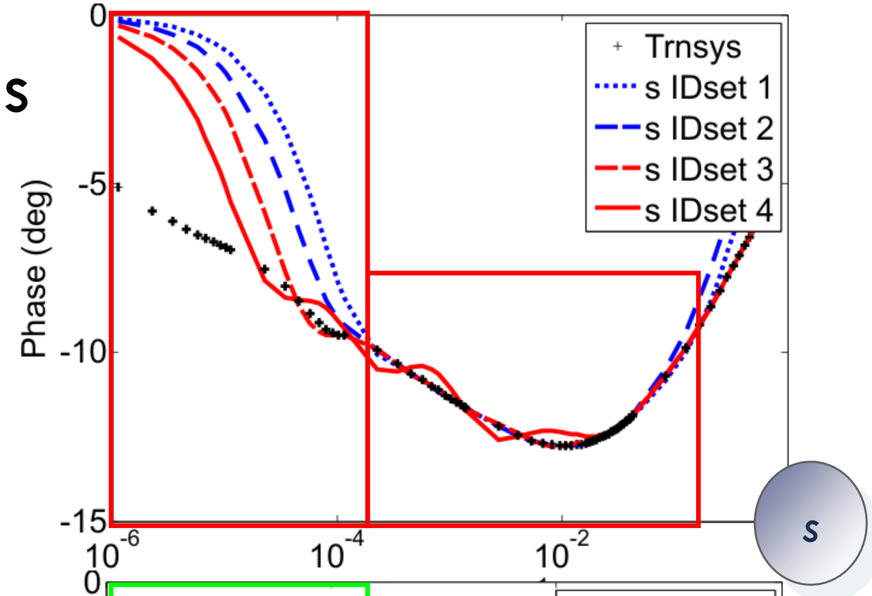


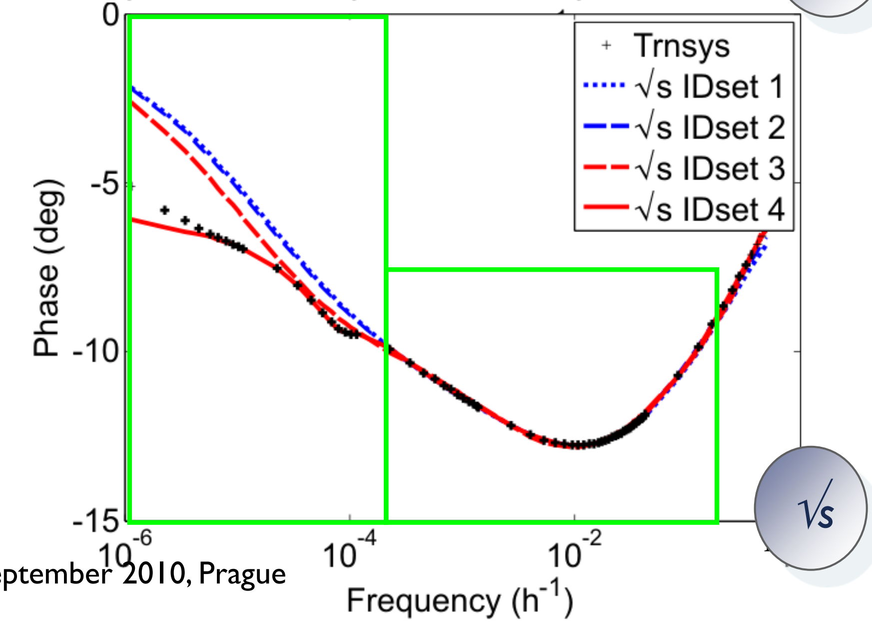
Ground coupled heat pump example

- 6. Extrapolation properties
  - Frequency domain



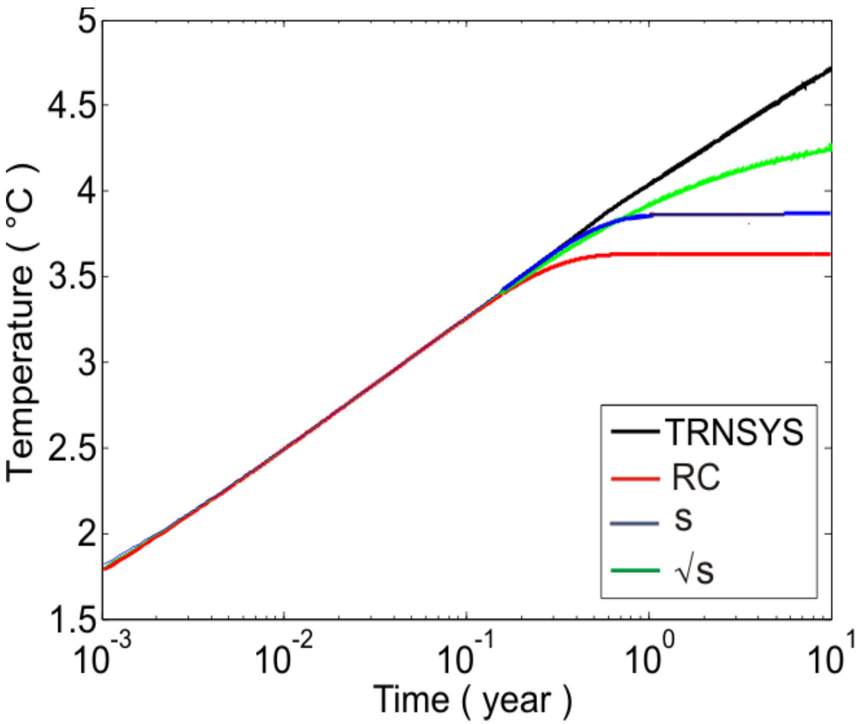
Extrapolation below frequencies ID





Ground coupled heat pump example

- 6. Extrapolation properties
  - Time domain



Response to step heat input

Ground coupled heat pump example

- 7. Model selection for MPC

RC-model	s-transfer function	√s-transfer function
✓ state space	✓ state space	X no state space
✓ excellent validation	X bad validation	✓ good validation
X no extrapolation	X no extrapolation	✓ good extrapolation

References

- Verhelst, C., Vandersteen, G., Schoukens, J., Helsen, L. (2009). **A linear dynamic borehole model for use in model based predictive control**. Proceedings of EFFSTOCK 2009. Effstock 2009. Stockholm, Sweden, 14-17 June 2009.
- Kollár, I. (2004-2007). **Frequency Domain System Identificaton Toolbox V3.3 for Matlab**, Gamax Ltd, Budapest
- Pintelon, R., Schoukens, J., Pauwels, L., Van Gheem, E., (2005). **Diffusion systems: stability, modeling and identification**, IEEE Transactions on Instrumentation and Measurement, vol.54, no.5, pp.2061-2067.

Applications in building control

- **Applications in building control**
  - Heating curve control
  - MPC for heavy-weight solar building
  - MPC for heat pump system with floor heating
  - MPC for ground coupled heat pump system
  - MPC for multizone building





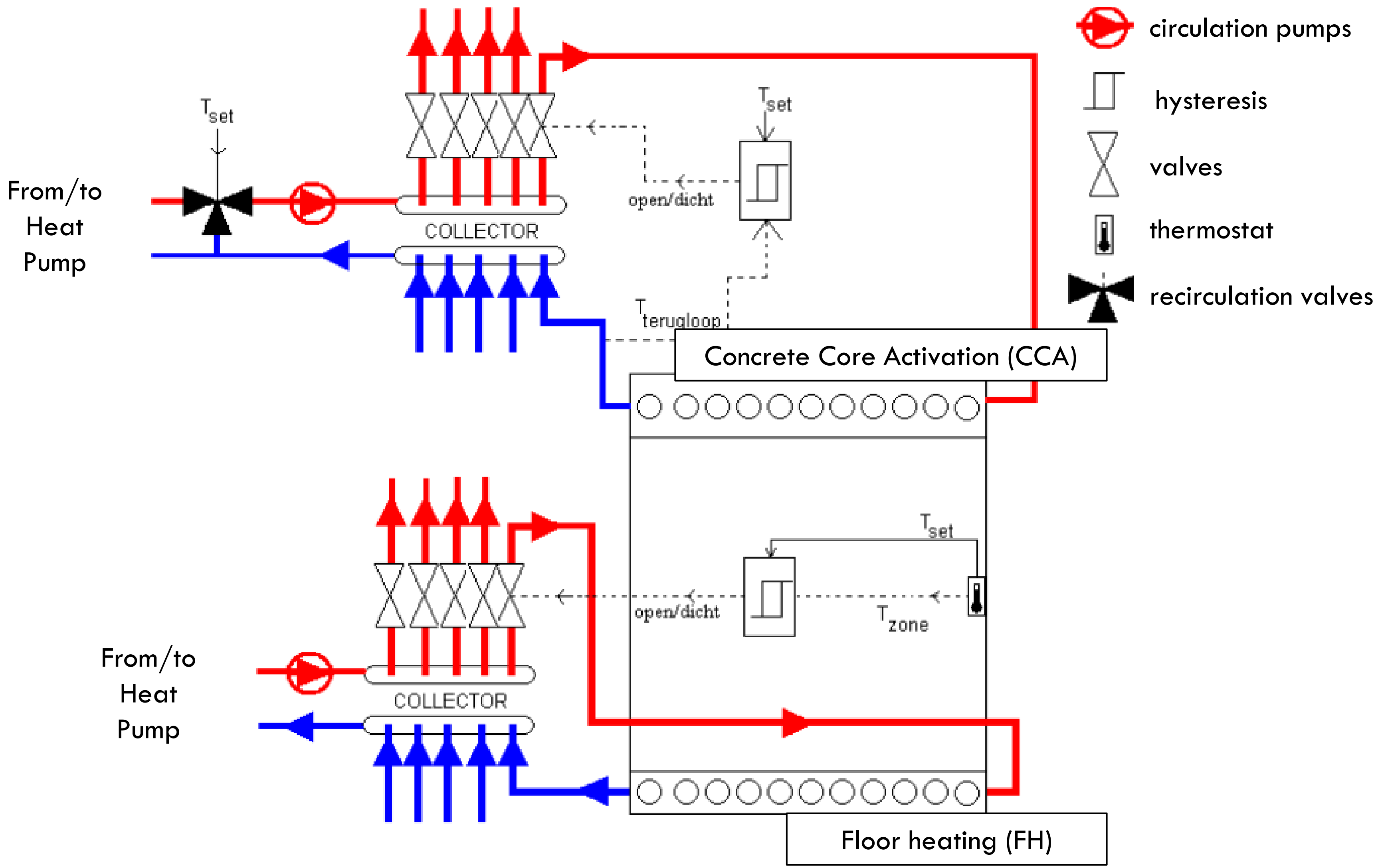
Multizone control

Pictures of the office building, Wellen, Belgium, used as case study

Multizone control

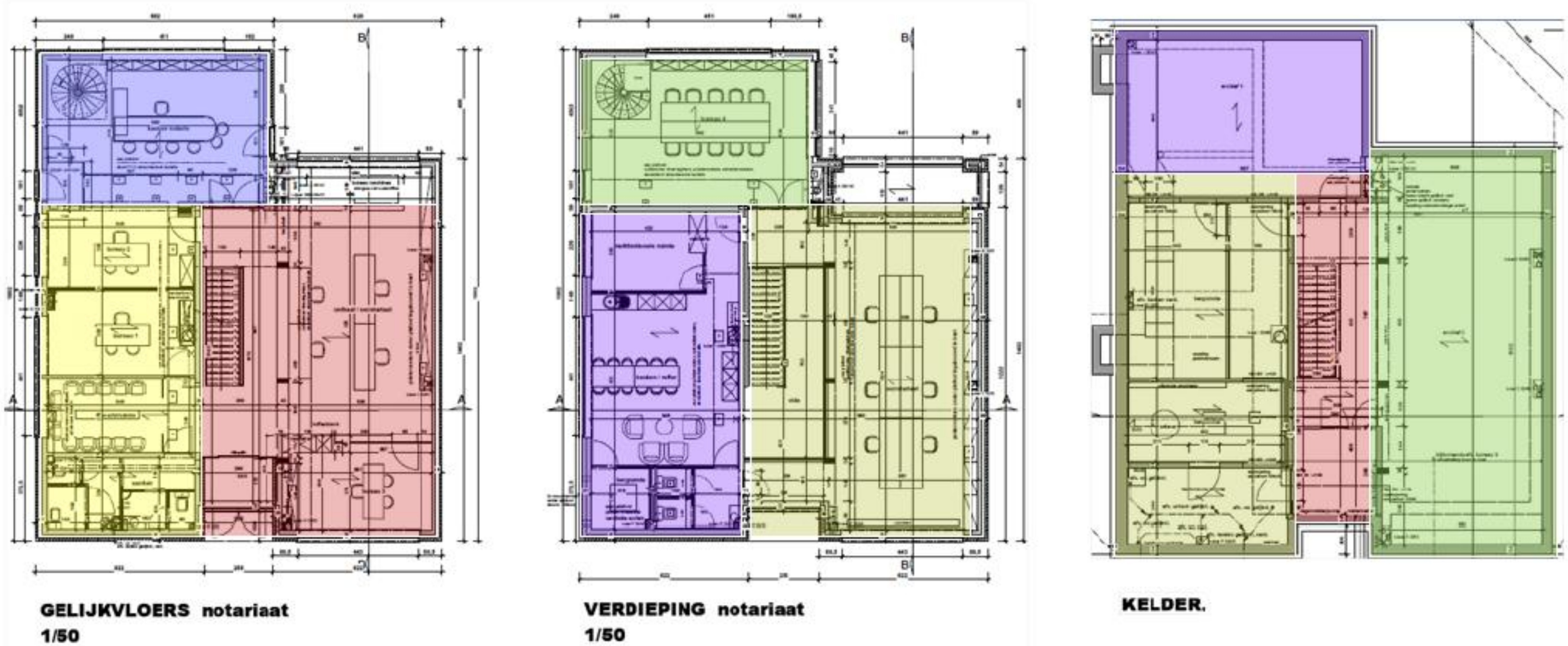
Valves of circuits towards concrete-core-activation of different zones

Multizone control



Multizone control

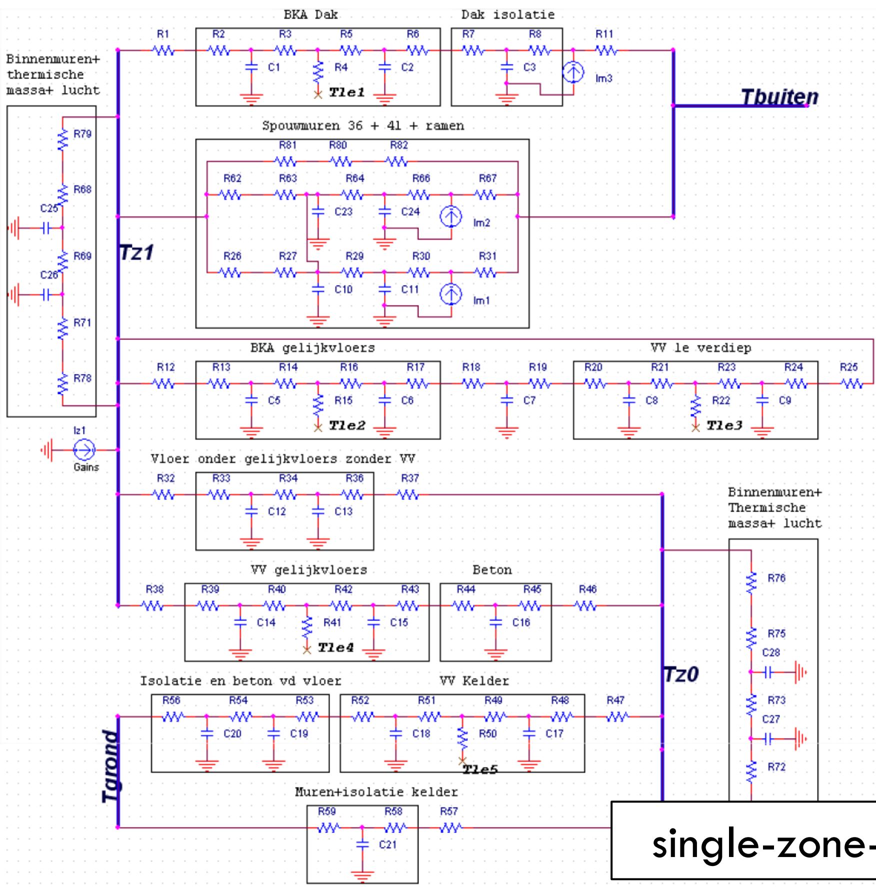
- Simulation model
  - Trnsys building simulation software



10-zone-simulation model

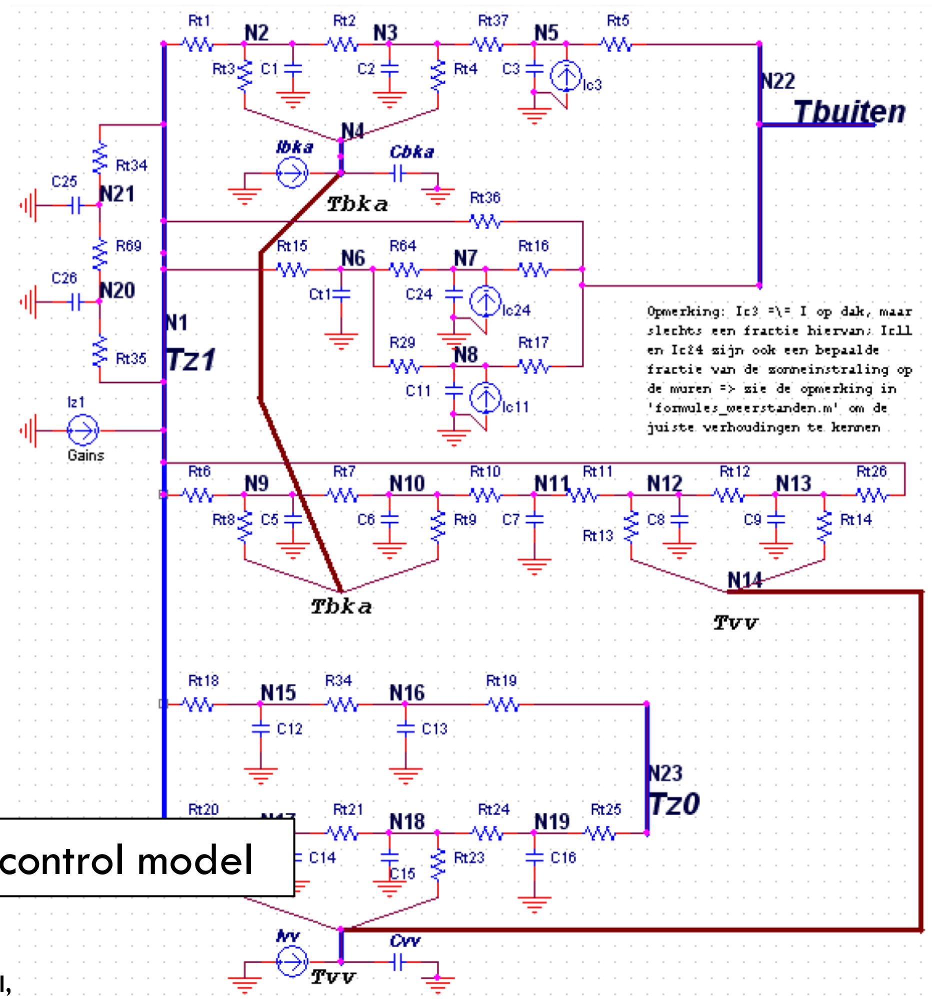
Multizone control

- Physical model for MPC



Figuur 40: Volledig RC-schema

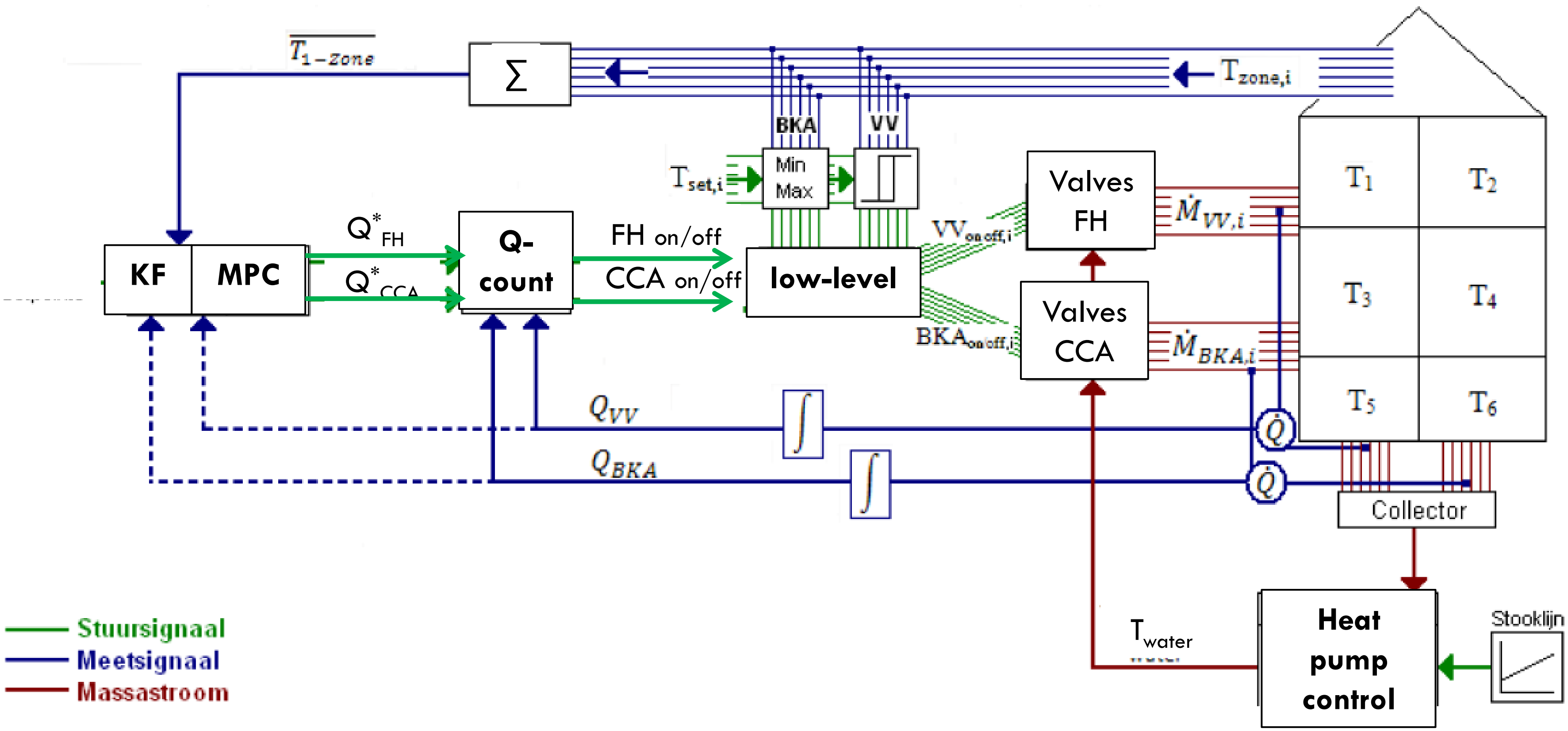
De weerstands- en capaciteitswaarden die horen bij dit schema zijn te vinden in Bijlage 2.



Figuur 41: Vereenvoudigd RC-schema

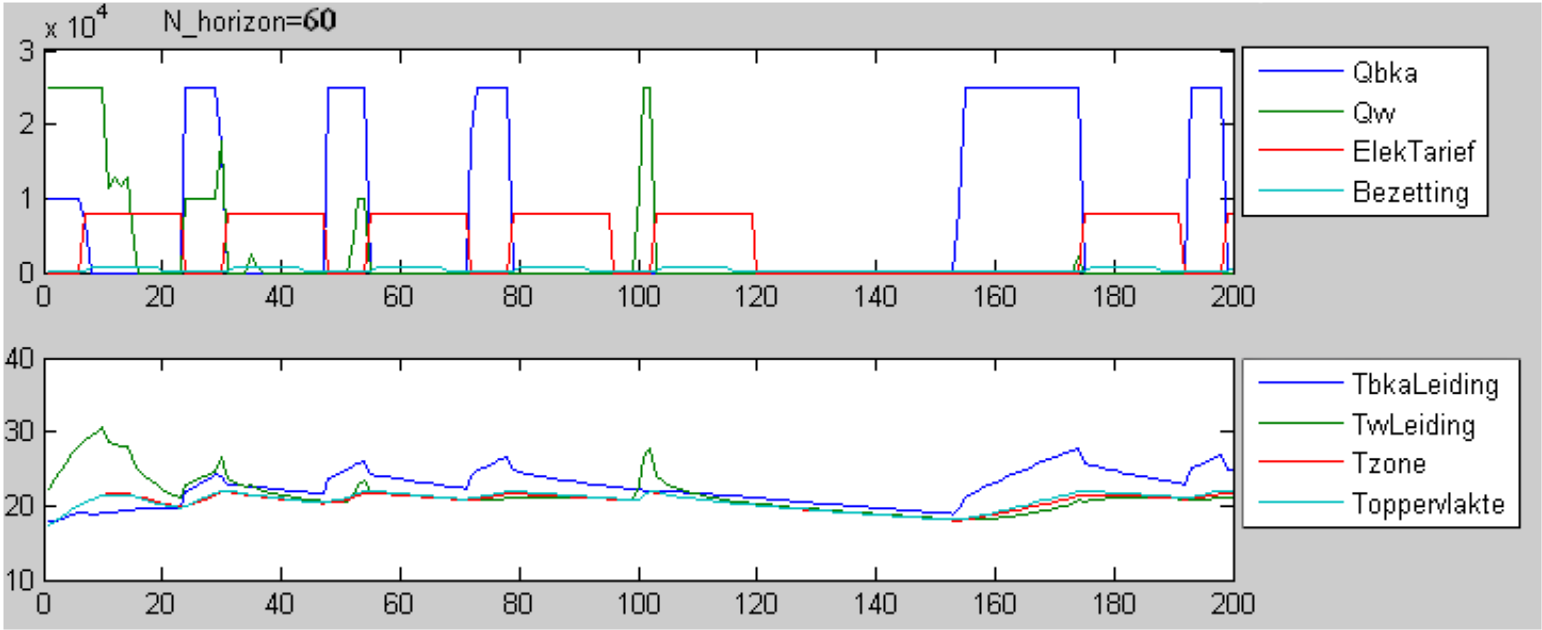
single-zone-control model

- Control scheme



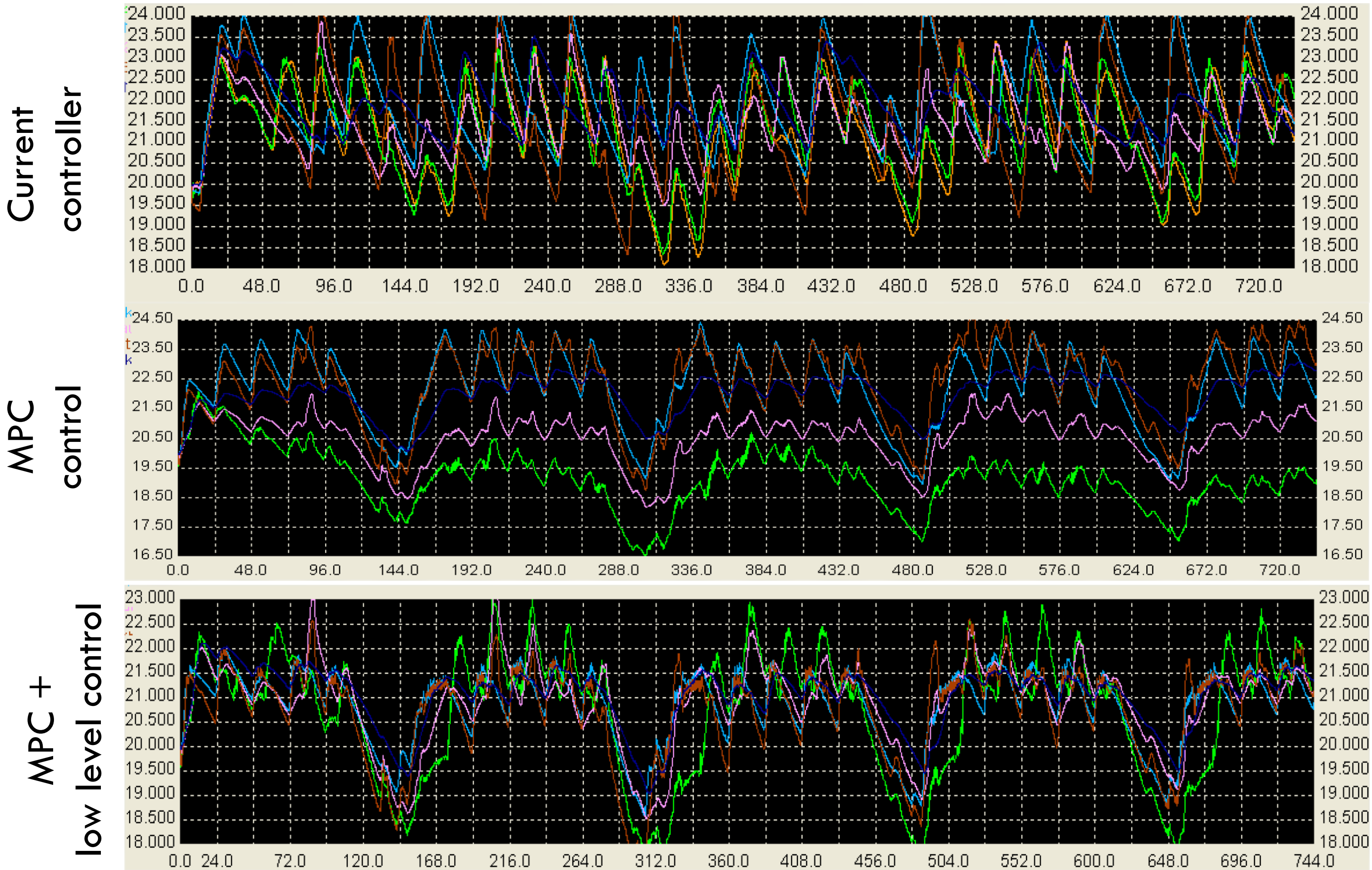
Multizone control

- Control performance evaluation



Upper: Optimal heat input profile for concrete-core activation (blue) and floor heating (green), given a day-night electricity rate tariff (red) and occupancy profile (cyan)
Lower: Calculated optimal supply water temperatures (blue and green) and zone temperature (red)

Multizone control



Multizone control

- Control performance evaluation

	Mean PPD (-)	Energy cost (€)	Electricity consumption (kWh)	Heat production (kWh)	COP (-)
Current control	5.1	257	1156	5806	5.0
MPC without low-level control	6.6	133	976	5070	5.2
MPC with low-level control	5.1	158	1023	5413	5.3

Multizone control

- Trade-off between
  - Model accuracy
  - Cost monitoring
- This case: Despite huge simplifications
  - Multizone → single zone
  - No ventilation losses
  - No internal gains
  - No solar gains
- ... still an improvement in control performance compared to conventional controller!

References

- Bax P.J., Krishnasing Y., 2010, **Notarisgebouw in Wellen: modellering, experimentele validatie en ontwikkeling MPC-regeling**, Thesis Master Energy, K.U.Leuven

Outline

- Introduction
- Framework of Model Predictive Control
- Development of control relevant model
- Applications in building control
- Conclusion

Conclusion

- Model information
  - Physical foundation makes sense!
- Model structure
  - Simplify as much as possible (but not more)
- Parameter estimation
  - Noise models
  - Excitation signal
- Model selection
  - Analyse control performance
  - Even with simplified models, significant improvement of control performance possible